PROFESSOR: OK. So we have set up the problem. We now have to understand what is the physics of this f of theta and phi. In a way, solving this scattering problem means solving for the thetas and phis. Now, you would say, OK, that seems reasonable. You know how the solution looks far away? And that's where you're going to do the measurements. That's where you have the detectors. So we have to find what f of theta and phi is. But suppose you had it. What have you learned if you have the f of theta and phi?

So this is what we need to compute, and we will build towards the computation of this using what is called partial waves-- waves and phase shifts. This will allow us to calculate the f of theta and phi. But doesn't tell you yet what f of theta and phi is. So the nice thing of this f of theta and phi is that it gives you the cross section of the process. So we need to understand what is the cross section.

So what is a cross section? It's a way to quantify the effect of the scattering center on the particles. Suppose you have your scattering center and you shoot particles in. Then you can say, OK, let me-- let's see with my detectors how many particles go off at an angle theta? When you say three particles per second, I'm finding as you should in particles, I'm getting three particles per second in this detector at this angle. My detector at this angle covers a solid angle of 100, and it's getting three particles per second.

Then you can say, OK, what is the area of a target that out of the incoming beam captures three particles per second? And maybe-- there are lots of particles coming in, but if you produce some area, that area will capture three particles per second. And that would be what we call the differential cross section. The value of the area that captures or just whose flux of particles is precisely what you're getting out there. So if you're looking at some process in which there's this incoming particles and you look at some angle-- d omega-- where there's a detector, a detector that capture this angle, you can associate to this object and d sigma-- which is a differential cross section-- with units of area.

And the physical interpretation of the d sigma for this-- the omega-- is that sigma is the area that captures the amount of flux that you see going in this direction. So it's a way this area-- this differential cross sections-- give you an iv, or a concrete representation, of how big the target is as seen by the particles that are coming in. I'll write it in a way that makes it clear.

So d sigma, which is called the differential cross section-- differential cross section-- is equal number of particles-- there's lots of words here, but it's good to write them-- scattered per unit time into the solid angle-- d omega-- divided by the flux of incident particles, which is equal to the number of particles per unit area per unit time.

So it's this ratio. Number of particles scattered per unit time into solid angle, d omega-- particles have no units. Integers scatter per unit time is 1 over time into the solid angle-- d omega has no units, either. The solid angle floods of incident particles is-- particles that has no units over area over time.

So here you had over time, here over time, they cancel, this is 1 over area. That ratio, therefore, has units of area. And this is what I was telling you. The differential cross section, which is an area, multiplied by the flux, gives you the number of particles per unit time that are crossing at differential cross section area. And those are set equal to the number of particles per unit time that end up within this solid angle.

So for a given little solid angle, you get the d sigma, which is small, as well. That's why it's called differential cross section. So let's try to show that this d sigma is really determined by f of theta and phi. This angle-- solid angle, d omega-- is happening at some theta and some phi. That's a position, the center of the little solid angle. A solid angle, d omega, at theta phi. So this is our goal now, to just calculate the differential cross section.

OK. So we have to compute this fluxes. We can do it here. Now, we're not working with wave pockets. We're working with energy eigenstates, and we're going to have a little bit of a fun intuition. We're not-- we're going to get it right, but you have to appreciate it's a little funny. If I have a little volume, and I say that I find the probability to find the particle there to be 1/2, I would say that there's a half of a particle in that little volume. If I have a volume that whose probability to find a particle is 1, I say, OK, I have 1 particle in here. That's the rough intuition.

So for wave functions that are not normalizable, it's almost like-- suppose size squared is equal to 1. In a momentum, I can say you would be saying like you have a particle for every unit volume where size squared integrates to 1. So that's roughly the intuition. And it's correct, really, to use it there. If you think of the flux of incident particles-- so incident flux-- flux-- we think of it as the probability current, which is number of particles per unit time per unit area. So it's h bar over m, imaginary part of the incident flux. So I should use the incident wave function, this phi of r, gradient phi of r.

And phi was e to the ikz. That was the incident particle. So this Laplacian gives you-- not Laplacian-- this gradient takes a gradient, it produces an ik, the rest gets canceled, and the imaginary part gives you k. So this is h bar k over m times the unit vector z. So that's the incident flux. This can be thought as the number of particles per unit area and per unit time.

You can also think of it intuitively as rho times v. The incident flux is like the incident current, the current density, it's rho times v. rho is phi squared-- phi squared. But that's equal to 1. And the velocity of the particles is the momentum over the mass. But that's hk over m. And if I put the direction, I will have to put the z. So it's kind of the same thing, rho v, the incident flux are all there. So this is the denominator of that big formula we have there-- the incident flux.

Then we need the number of particles scattered per unit time. We could do it with a flux calculation. Also, in spherical coordinates, but I can also do it intuitively. So I'll do it intuitively. You can try doing it using a probability current. So intuitively what do we have? We have a d omega here. And let's consider a little volume element here.

So a little volume element. Here you have r-- distance r-- a little vr. So a small volume element here. And let's calculate how many particles there are in this small little element. So dn is number of particles in the little volume-- volume. So I must square the wave function and multiply by the volume. That's what you would do.

So what is the square of the wave function? Now, we are talking about the scattered wave function. So I must take this part of the wave function and square it and multiply by the volume. So we will have f of theta and phi times e to the ikr over r, all squared. So that's the wave function squared.

That's psi carat squared times the air-- volume of this pill box is r squared d omega times dr. r squared d omega is the surface area of that little box. dr is the little length there. And, therefore, we have that volume over there. So things simplify the n is equal, therefore, the r squared cancels, and it's just f of theta phi squared, d omega, dr.

So with all these particles-- this is the number of particles inside the box. But all these particles will go through in a little time dt, which is equal to dr over the velocity. This is the time to go through the box-- through the box. dt is dr over the velocity. And the velocity is hk over m. So dr times the momentum over the mass, which is hk over m. OK. So we have dn, dt. This is number of particles. We divide them to form number of particles per unit time. So the ratio dn dt is the numerator of this quantity here-- is numerator dn [? dt. ?]

And what do we get? The dr cancels and we're dividing the ndt-- this factor goes into the numerator, so we get h bar k over m f of theta phi squared d omega. So this is the numerator of that equation for the d sigma. So let's compute-- finish that. We finally-- I'm going to get off our identification. And what do we get? We get d sigma is the numerator there, which is this quantity, h bar k over m f of theta phi squared times d omega over h bar k over m, which is nice. That all going to cancel.

And then we get the nice formula that we were after. This differential cross section is just determined by the function f of theta phi that we need to calculate. f of theta phi is our goal. If we have it, we have the differential cross section. Many people write this formula this way-- d sigma, d omega is equal to f of theta phi. I think that's OK. I think in many ways this is a little clearer. This small little area associated to a small little angle is given by that. This is a ratio of differentials more than a derivative. A derivative with respect to solid angle doesn't mean too much, I think.

So-- but this is another way people write it. And, finally, people integrate this total cross section is the integral of the differential cross section over-- looking at all solid angles. So this means integrating f of theta phi squared d omega. And that's the total cross section.

OK. So we've set up the problem. We-- what are the main things that we've learned? We have the scattering center. We have our physical condition of a wave coming in. Our physical condition that the wave comes out. Those are represented here.

And this situation is saying that this is an approximate solution for a wave-- a wave coming out modulated by a theta and phi dependence and your incoming wave. This modulation is the thing that captures the effect of the potential and associates to the strength of this potential and its ability to scatter a differ-- a cross section, which is measurable-- which means a probability of interaction-- capture the probability of interaction.

This cross sections are very important. When you design an experiment with an accelerator, you want to have a cross section is sufficiently large, because this cross section is going to tell you how many particles your detector is going to get. Whether with the flux that you have, with the beam intensity that you have, you're going to get one Higgs a day, or 500 Higgs's a day. And it made a difference for the LHC. It began with a few Higgs's a day, and later, it's getting a few hundred Higgs's a day. So-- so those quantities are pretty important.