8.044 SOLUTIONS PRACTICE EXAM #2

1. EXPAND $d\mathcal{F} = \left.\frac{\partial \mathcal{F}}{\partial T}\right|_L dT + \left.\frac{\partial \mathcal{F}}{\partial T}\right|_L dL$

FIND COEFFICIENTS $\left.\frac{\partial \mathcal{F}}{\partial L}\right|_T = (a + bT)$ IS GIVEN

$$\left.\frac{\partial \mathcal{F}}{\partial T}\right|_L = -1 \Big/ \left.\frac{\partial T}{\partial L}\right|_{\mathcal{F}} \left.\frac{\partial L}{\partial \mathcal{F}}\right|_T = -\left.\frac{\partial L}{\partial T}\right|_{\mathcal{F}} \left.\frac{\partial \mathcal{F}}{\partial L}\right|_T = b\,(L - L_0)$$

SO $d\mathcal{F} = b(L - L_0)\,dT + (a + bT)\,dL$

$$\mathcal{F} = b(L - L_0)T + f(L)$$

$$\left.\frac{\partial \mathcal{F}}{\partial L}\right|_T = bT + f'(L) \overset{\text{ALSO}}{=} a + bT \Rightarrow f'(L) = a, \quad f(L) = aL + C$$

$\mathcal{F} = b(L - L_0)T + aL + C$ MUST $= 0$ AT $L = L_0$ $\Rightarrow C = -aL_0$

$$\underline{\mathcal{F} = (a + bT)(L - L_0)}$$

2. $đQ = dU - đW = dU - H\,dM$ 1^{ST} LAW

$$= \left.\frac{\partial U}{\partial T}\right|_M dT + \left(\cancelto{0 \text{ BY EXPERIMENT}}{\left.\frac{\partial U}{\partial M}\right|_T} - H\right) dM \qquad \text{EXPANSION OF } U$$

$C_M \equiv \left.\frac{đQ}{dT}\right|_M = \left.\frac{\partial U}{\partial T}\right|_M$ IS A CONSTANT BY EXPERIMENT

ADIABATIC $\equiv đQ = 0 = C_M\,dT - \left(\frac{MT}{a}\right) dM$

$$\frac{dT}{T} = \frac{1}{aC_M} M\,dM \Rightarrow \ln\frac{T}{T_0} = \frac{1}{2aC_M}\left(M^2 - M_0^2\right)$$

$$\frac{T}{T_0} = e^{\frac{M^2 - M_0^2}{2aC_M}} \qquad \underline{T_{FINAL}/T_0 = e^{-M_0^2/2aC_M}}$$

3. a/ Ω = # WAYS OF PLACING n ATOMS IN N SURFACE CELLS
X # WAYS OF PLACING $N-n$ ATOMS IN M BULK CELLS

$$= \frac{N!}{(N-n)!\,n!}\;\frac{M!}{(M-(N-n))!\,(N-n)!}$$

$$S(n) = k\ln\Omega = k\Big\{N\ln N - (N-n)\ln(N-n) - n\ln n + M\ln M - (M-N+n)\ln(M-N+n) - (N-n)\ln(N-n)\Big\}$$

b/ $$\frac{1}{T} = \frac{\partial S(E)}{\partial E}\Big|_{N,M} = \frac{\partial S}{\partial n}\frac{\partial n}{\partial E} = \frac{\partial S}{\partial n}\left(\frac{-1}{\epsilon}\right)$$

$$= \frac{-k}{\epsilon}\Big\{\ln(N-n)+1-\ln n-1-\ln(M-N+n)-1+\ln(N-n)+1\Big\}$$

$$= \frac{-k}{\epsilon}\ln\left[\frac{(N-n)^2}{n(M-N+n)}\right]$$

$$\underline{\frac{(N-n)^2}{n(M-N+n)} = e^{-\epsilon/kT}}$$

c/ $e^{-\epsilon/kT} = 0$ AT $T=0$ $\Rightarrow$ $\underline{n=N}$ $\Big\{$ ALL THE ATOMS ARE ON THE SURFACE

d/ $e^{-\epsilon/kT} = 1$ AT $T=\infty$

$$\Rightarrow N^2 - 2nN + n^2 = nM - nN + n^2 \Rightarrow N^2 = (M+N)n$$

$$\underline{\frac{n}{N} = \frac{N}{M+N}}$$ $\Big\{$ FRACTIONAL OCCUPATION OF SURFACE STATES IS IDENTICAL TO THE FRACTIONAL OCCUPATION OF ALL THE STATES.

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