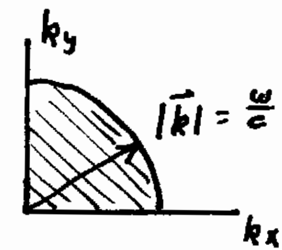
8.044 SOLUTIONS, EXAM #4 SPRING 2004

1. a) $\sin k_x L = 0 \Rightarrow k_x = n_x \frac{\pi}{L}$ $\quad n_x = 1, 2, 3, 4 \cdots$

$\sin k_y L = 0 \Rightarrow k_y = n_y \frac{\pi}{L}$ $\quad n_y = 1, 2, 3, 4 \cdots$

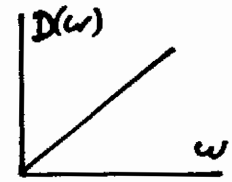
$$\underline{\vec{k} = \frac{\pi}{L}\left(n_x \hat{x} + n_y \hat{y}\right)}$$

b) $D(k) = \left(\frac{L}{\pi}\right)^2 \qquad \omega = c|\vec{k}|$



$\#(\omega) = \left(\frac{L}{\pi}\right)^2 \frac{1}{4}\pi\left(\frac{\omega}{c}\right)^2$ $\Big\{$ ONLY ONE POLARIZATION FOR $\vec{E}$ IN THIS WORLD

$$\underline{D(\omega) = \frac{d\#}{d\omega} = \frac{1}{2\pi}\left(\frac{L}{c}\right)^2 \omega}$$



c) $u(\omega, T) = \frac{1}{L^2} D(\omega) \langle \epsilon(\omega) \rangle = \frac{1}{L^2} D(\omega)\, \hbar\omega \langle n(\omega, T)\rangle_{\text{HARMONIC OSCILLATOR}}$

$$\underline{= \frac{\hbar\omega^2}{2\pi c^2} \frac{1}{e^{\hbar\omega/kT} - 1}}$$

d) $u(T) = \int_0^\infty u(\omega, T)\, d\omega = \frac{1}{2\pi\hbar^2 c^2}(kT)^3 \underbrace{\int_0^\infty \frac{x^2\, dx}{e^x - 1}}_{\text{DIMENSIONLESS}}$

$e(T) \propto u(T)$ FOR A BLACK BODY

$\underline{e(T) \propto T^3 \text{ IN THIS WORLD}}$ FOR BLACK BODY EMISSION

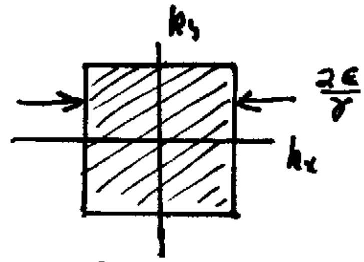
2. a) $e^{ik_x(x+L)} = e^{ik_x x}\underbrace{e^{ik_x L}}_{1} = e^{ik_x x}$

$\Rightarrow k_x = n_x \frac{2\pi}{L} \quad n_x = 0, \pm 1, \pm 2 \cdots$

SAME FOR k_y

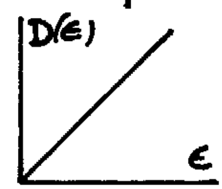
$\underline{\vec{k} = \frac{2\pi}{L}(n_x \hat{x} + n_y \hat{y}) \quad n_i = 0, \pm 1, \pm 2 \cdots}$

b) $\underline{D(\vec{k}) = \left(\frac{L}{2\pi}\right)^2 \text{ FOR ALL } \vec{k}}$



c) $\#(\epsilon) = \overset{\text{SPIN}}{2}\, D(\vec{k}) \left(\frac{2\epsilon}{\gamma}\right)^2$

$D(\epsilon) = 2\left(\frac{L}{2\pi}\right)^2 \frac{8}{\gamma^2}\epsilon = \underline{4\left(\frac{L}{\pi\gamma}\right)^2 \epsilon}$



d) $N = \#(\epsilon_F) = \frac{8L^2}{(2\pi)^2}\left(\frac{\epsilon_F}{\gamma}\right)^2$

$\underline{\epsilon_F = \left(\frac{1}{2}\pi^2\gamma^2\left(\frac{N}{A}\right)\right)^{1/2}}$

e) $D(\epsilon) = \alpha\epsilon \qquad N = \int_0^{\epsilon_F} \alpha\epsilon\, d\epsilon = \frac{\alpha}{2}\epsilon_F^2$

$E = \int_0^{\epsilon_F} \alpha\epsilon^2 d\epsilon = \frac{\alpha}{3}\epsilon_F^3 = \underline{\frac{2}{3}N\epsilon_F}$

f) $\underline{\text{STILL } T^1}$

g) $dE = \overset{0 \text{ AT } T=0}{\cancel{T dS}} + \mathcal{S}\, dA$

$\mathcal{S} = \frac{dE}{dA} = \frac{2}{3}N\frac{d\epsilon_F}{dA} = \frac{2}{3}N\left(-\frac{1}{2}\right)\frac{\epsilon_F}{A} = \underline{-\frac{1}{3}\left(\frac{N}{A}\right)\epsilon_F}$

3. a) $Z_1 = e^{\mu_0 H/kT} + 2 + e^{-\mu_0 H/kT} = \underline{2\left(1+\cosh(\mu_0 H/kT)\right)}$

b) $$E = N\langle\epsilon\rangle = N\,\frac{-\mu_0 H e^{\mu_0 H/kT} + 0 + \mu_0 H e^{-\mu_0 H/kT}}{Z_1}$$

$$= \underline{-\mu_0 H N\,\frac{\sinh(\mu_0 H/kT)}{1+\cosh(\mu_0 H/kT)}}$$

c) $$M = N\langle\mu_z\rangle = N\,\frac{\mu_0 e^{\mu_0 H/kT} + 0 - \mu_0 e^{-\mu_0 H/kT}}{Z_1}$$

$$= \underline{\mu_0 N\,\frac{\sinh(\mu_0 H/kT)}{1+\cosh(\mu_0 H/kT)}}$$

8.044 Statistical Physics I
Spring 2013