2 a) $$p(E_B) = \int_{-\infty}^{\infty} p(E_A, E_B)\, dE_A$$

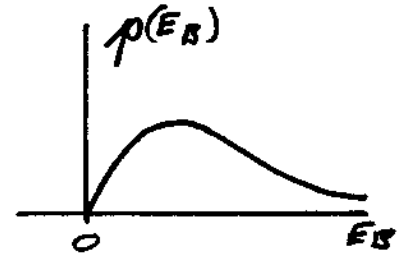
$$= \frac{4E_B}{\Delta^4} e^{-E_B/\Delta} \int_{E_B}^{\infty} E_A e^{-E_A/\Delta}\, dE_A$$

$$- \frac{4E_B^2}{\Delta^4} e^{-E_B/\Delta} \int_{E_B}^{\infty} e^{-E_A/\Delta}\, dE_A$$

$$= \frac{4E_B}{\Delta^2} e^{-E_B/\Delta} \left(1 + \frac{E_B}{\Delta}\right) e^{-E_B/\Delta}$$

$$- \frac{4E_B^2}{\Delta^3} e^{-E_B/\Delta} e^{-E_B/\Delta}$$

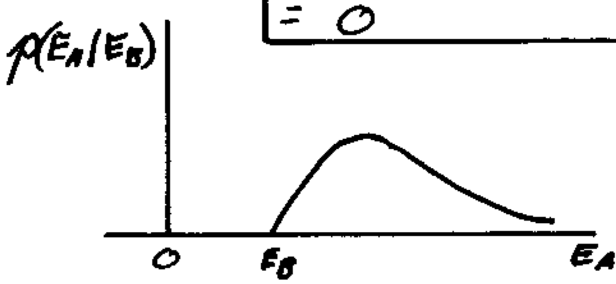
$$\underline{\underline{= \frac{2}{\Delta} \left(\frac{2E_B}{\Delta}\right) e^{-2E_B/\Delta} \qquad E_B > 0}}$$



b) $$p(E_A | E_B) = p(E_A, E_B) / p(E_B)$$

$$= \frac{1}{\Delta} \left(\frac{E_A - E_B}{\Delta}\right) e^{-(E_A - E_B)/\Delta} \qquad E_A > E_B$$

$$= 0 \qquad \text{ELSEWHERE}$$



c) NOT S.I. BECAUSE $p(E_A | E_B)$ DEPENDS ON E_B.

d) POISSON PROCESS $\langle n \rangle = f \times 10^6 h$ $\quad h \equiv$ TIME IN HOURS

REQUIRING $\sqrt{\mathrm{Var}(n)} / \langle n \rangle = 10^{-4} \Rightarrow \sqrt{\langle n \rangle} = 10^4$

$\Rightarrow \langle n \rangle = 10^8 = f \times 10^6 h$

$$\underline{\underline{h = \frac{100}{f} \text{ HOURS}}}$$

8.044 SOLUTIONS EXAM #2

1. $P = P(T,\varepsilon) \Rightarrow dP = \frac{\partial P}{\partial T}\Big|_\varepsilon dT + \frac{\partial P}{\partial \varepsilon}\Big|_T d\varepsilon$

$$\frac{\partial P}{\partial \varepsilon}\Big|_T = \left(a + \frac{b}{T}\right)N + 3cN\varepsilon^2 \quad \{\text{GIVEN}\}$$

$$\frac{\partial P}{\partial T}\Big|_\varepsilon = \frac{-1}{\frac{\partial T}{\partial \varepsilon}\Big|_P \frac{\partial \varepsilon}{\partial P}\Big|_T} = -\frac{\frac{\partial P}{\partial \varepsilon}\Big|_T}{\frac{\partial T}{\partial \varepsilon}\Big|_P} \quad \{\text{GIVEN}\}$$

$$= \frac{\left[\left(a+\frac{b}{T}\right)N + 3cN\varepsilon^2\right]\left[dT^2 - b\varepsilon\right]}{aT^2 + bT + 3cT^2\varepsilon^2} = \frac{N}{T^2}\left[dT^2 - b\varepsilon\right]$$

$$= Nd - \frac{bN\varepsilon}{T}$$

$$P = \left(a + \frac{b}{T}\right)N\varepsilon + cN\varepsilon^3 + f(T) \quad \left\{\begin{array}{l}\text{INTEGRATING FIRST}\\ \text{WITH RESPECT TO } \varepsilon\end{array}\right\}$$

$$\frac{\partial P}{\partial T}\Big|_\varepsilon = -\frac{bN\varepsilon}{T^2} + f'(T) = Nd - \frac{bN\varepsilon}{T^2} \Rightarrow f'(T) = Nd$$

$$f(T) = NdT + K$$

$$P(T_0, \varepsilon = 0) = P_0 = NdT_0 + K \Rightarrow K = P_0 - NdT_0$$

$$\underline{\underline{P = \left(a + \frac{b}{T}\right)N\varepsilon + cN\varepsilon^3 + Nd(T - T_0) + P_0}}$$

3 a/ COMPUTE THE CUMULATIVE VOLUME IN PHASE SPACE FIRST.

$$\phi = \left[\int_0^{2\pi} d\theta\right]^N \int_{\sum_{i=1}^N \frac{l_i^2}{2I} < E} \{dl_i\} = (2\pi)^N \times \text{VOLUME OF N DIMENSIONAL SPHERE OF RADIUS } \sqrt{2IE}$$

$$= (2\pi)^N \pi^{N/2} (2IE)^{N/2} \frac{1}{(N/2)!}$$

$$\approx (2\pi)^N (2\pi IE)^{N/2} \frac{1}{\left(\frac{N}{2e}\right)^{N/2}} = (2\pi)^N \left(\frac{4\pi e I E}{N}\right)^{N/2}$$

$$\Omega = \Delta \frac{\partial\phi}{\partial E} = \underline{\underline{\left(\frac{N\Delta}{2E}\right)(2\pi)^N \left(\frac{4\pi e I E}{N}\right)^{N/2}}}$$

b/ $p(\theta) = \Omega'(\text{ONE ANGLE FIXED})/\Omega = \frac{(2\pi)^{N-1}}{(2\pi)^N}$

$$= \underline{\underline{\frac{1}{2\pi} \quad 0 \le \theta < 2\pi, \quad = 0 \text{ ELSEW.}}}$$

c/ Ω': SET $N \to N-1$, $E \to E - \frac{l^2}{2I}$ IN ANGULAR MOMENTUM CONTRIBUTION

$$\underline{\underline{\Omega' = \left(\frac{(N-1)\Delta}{2(E - l^2/2I)}\right)(2\pi)^N \left(\frac{4\pi e I (E - l^2/2I)}{N-1}\right)^{(N-1)/2}}}$$

E WILL BE REPLACED BY $\underline{\underline{N\langle l^2\rangle/2I}}$

d/ $S = k \ln\phi$

$$\frac{1}{T} = \left.\frac{\partial S}{\partial E}\right|_N = k \frac{1}{\phi} \left.\frac{\partial\phi}{\partial E}\right|_N = k \frac{1}{\phi}\left(\frac{N}{2}\frac{1}{E}\phi\right) = \frac{Nk}{2E}$$

$$\underline{\underline{E = \tfrac{1}{2}NkT}}$$

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Spring 2013