MASSACHUSETTS INSTITUTE OF TECHNOLOGY

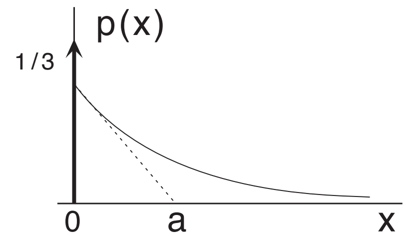
Physics Department

8.044 Statistical Physics I — Spring Term 2013

Problem Set #3

Due in hand-in box by 12:40 PM, Wednesday, February 27

Problem 1: Clearing Impurities



In an effort to clear impurities from a fabricated nano-wire a laser beam is swept repeatedly along the wire in the presence of a parallel electric field. After one sweep an impurity initially at $x = 0$ has the following probability density of being found at a new position x

$$p(x) = \frac{1}{3}\,\delta(x) + \frac{2}{3a}\exp[-x/a] \qquad 0 \leq x$$

$$= 0 \qquad \text{elsewhere}$$

where a is some characteristic length.

Give an approximate probability density for the total distance d the impurity has moved along the wire after 36 sweeps of the laser beam.

Problem 2: Probability Densities of Macroscopic verses Microscopic Variables

Consider one cubic centimeter of a dilute gas of atoms of mass M in thermal equilibrium at temperature T= 0^o C and atmospheric pressure. (Recall that Lochmidt's number – the number of atoms (or molecules) in a cubic meter of an ideal gas at T= 0^o C and atmospheric pressure – has the value $2.69\times 10^{25}\ m^{-3}$.)

a) For the kinetic energy of a single atom, find a numerical value for the ratio of standard deviation (the square root of the variance) to the mean. You may use the results you found in problem 4 on Problem Set 2.

b) Find the same ratio for total energy of the gas, assumed to be all kinetic.

Problem 3: Temperature

Systems A and B are paramagnetic salts with coordinates H, M and H', M' respectively. System C is a gas with coordinates P, V. When A and C are in thermal equilibrium, the equation

$$nRCH - MPV = 0$$

is found to hold. When B and C are in thermal equilibrium, we get

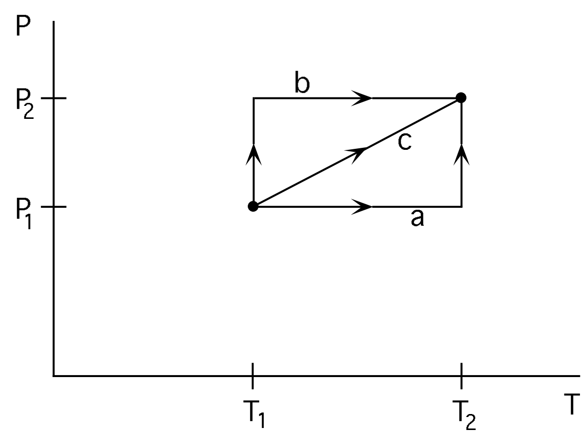
$$nR\Theta M' + nRC'H' - M'PV = 0$$

where n, R, C, C', and Θ are constants.

a) What are the three functions that are equal to one another at thermal equilibrium?

b) Set each of these functions equal to the ideal gas temperature T and see if you recognize any of these equations of state.

Problem 4: Work in a Simple Solid

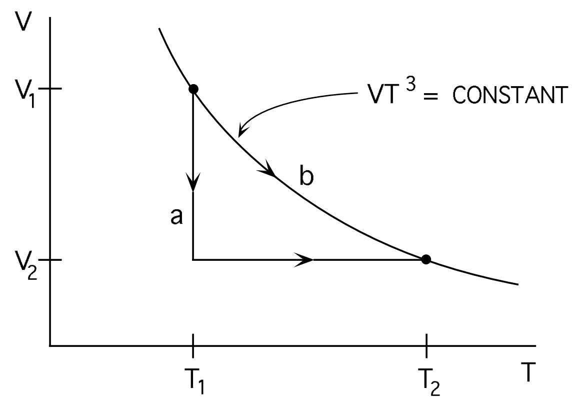


In the simplest model of an elastic solid

$$dV = -V\mathcal{K}_T dP + V\alpha dT$$

where $\mathcal{K}_T$ is the isothermal compressibility and α is the thermal expansion coefficient. Find the work done on the solid as it is taken between state (P_1, T_1) and (P_2, T_2) by each of the three paths indicated in the sketch. Assume that the fractional volume change is small enough that the function $V(P,T)$ which enters the expression for dV can be taken to be constant at $V = V_1 = V(P_1, T_1)$ during the process.

Problem 5: Work and the Radiation Field



The pressure P due to the thermal equilibrium radiation field inside a cavity depends only on the temperature T of the cavity and not on its volume V,

$$P = \frac{1}{3}\sigma T^4.$$

In this expression σ is a constant. Find the work done on the radiation field as the cavity is taken between states (V_1, T_1) and (V_2, T_2) along the two paths shown in the diagram.

Practice Problem, <u>do not hand this in</u>: Exact Differentials

Which of the following is an exact differential of a function $S(x, y)$? Find S where possible.

a) $2x(x^3 + y^3)dx + 3y^2(x^2 + y^2)dy$

$$S(x, y) = (2x^5 + 5x^2y^3 + 3y^5)/5 + C$$

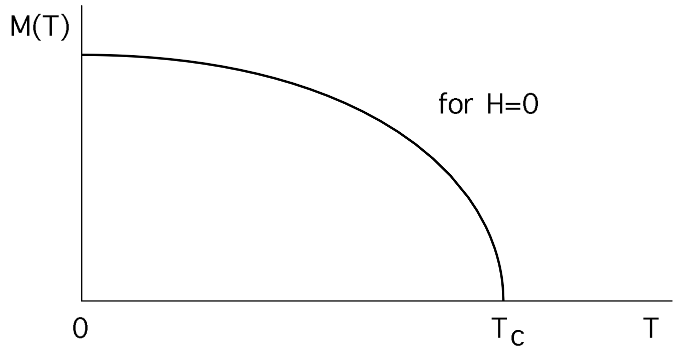
b) $e^y dx + x(e^y + 1)dy$

$S(x, y)$ does not exist.

c) $(y - x)e^x dx + (1 + e^x)dy$

$$S(x, y) = y + (1 + y - x)e^x + C$$

Problem 6: Equation of State for a Ferromagnet



For a ferromagnetic material in the absence of an applied field, $H = 0$, the spontaneous magnetization is a maximum at $T = 0$, decreases to zero at the critical temperature $T = T_c$, and is zero for all $T > T_c$.

For temperatures just below T_c the magnetic susceptibility and the temperature coefficient of M might be modeled by the expressions

$$\chi_T \equiv \left(\frac{\partial M}{\partial H}\right)_T = \frac{a}{(1 - T/T_c)} + 3bH^2$$

$$\left(\frac{\partial M}{\partial T}\right)_H = \frac{1}{T_c}\frac{f(H)}{(1 - T/T_c)^2} - \frac{1}{2}\frac{M_0}{T_c}\frac{1}{(1 - T/T_c)^{1/2}}$$

where M_0, T_c, a, and b are constants and $f(H)$ is a function of H alone with the property that $f(H = 0) = 0$.

a) Find $f(H)$ by using the fact that M is a state function.

b) Find $M(H, T)$.

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