PROFESSOR: So I want to go a little further to try to put resonances in a more intriguing footing. That you can play with and if you-- at some point interested.

So let's think of discovering [INAUDIBLE] that we have. We had A s-- remember the scattered wave was A s e to the ikx [INAUDIBLE] that divided 2. And what was A s? Well, A s squared-- the sine square delta. So if you remember this was sine delta e to the i delta. So let's stick to that and try to write it in a funny way. Certainly, A s is becoming large near resonance, so let's think when A s becomes large. Well, in another way let's be a little creative about things, It's good sometimes not to be logical.

So let's write this as sine delta-- I'll do it here-- sine delta over e to the minus i of delta . And that's sine delta over close delta minus i sine delta. That's all good. A s-- let me divide by sine delta both sides-- both numerator and denominator. So-- no divide it by cosine delta, so I'll have tan delta over 1 minus i tan delta. I divide it by cosine.

You want A s large? You really want it large, choose tan delta-- equals to minus i. Sounds crazy, but it's not really crazy. The reason it sounds crazy and it's somewhat strange and not very logical is tan delta is a phase and the tangent of any phase is never an imaginary number.

So then I would have think of delta itself as a complex number. And what would that mean. So things are weird. But it's certainly the fact that A s will become infinite-- not just large-- but infinite. A s will become infinite. And you say, wow, this doesn't make any sense.

But maybe it makes sense in the following way. This is the line of real phase shifts. [INAUDIBLE] are real. And here is the world of complex phase shifts. These are the real phase shifts and there are the complex phase shifts. Maybe if the phase shift becomes infinite-- off the real axis-- it's just large on the real axis. So actually, if you wanted it to be very large you would have to get off the real axis. If this sounds vague, it is still vague. But in a minute we'll make it precise.

So I suggest that we take this idea seriously-- that maybe this means something. And we can try to argue that by looking back at what resonances do. So what I will do is look with [INAUDIBLE] a resonance here-- tangent delta. So let's look at what A s does. We have it there. A s is tan delta-- well, tan delta-- we had it in the middle of blackboard is beta over alpha minus k, 1 minus i beta over alpha minus k, again.

So that's how A s behaves in general. That's fine, there's no -- at this moment there's nothing crazy about this. Because this is something you all agreed, nobody complained about this formula. So A s is given by that formula-- that's also legal math, so far. So we'll have this. And then let's simplify it a little bit which is beta over alpha minus k minus i beta. So this still beta over alpha minus i beta minus k.

So we usually would plot A s as a function of k. That's what we're trying to do, it's a function of k. And now here is the formula for A s as the function of k. And here is k. But let's be daring now and not say this is k, this is the complex k-plane. And yes, you work with real k, but that's because that has a direct physical interpretation. But maybe the complex plane has a more subtle physical interpretation and that's what they claim is happening here.

This quantity becomes infinite near the resonance. Here was the resonance, what you call the resonance. But this becomes really infinite not at alpha-- for when k is equal to alpha, but when k is equal to alpha minus i beta. Beta was supposed to be small for a resonance. So here is minus i beta and here is this very unusual point. Where the scattering amplitude blows up. It has what is in complex variables-- if you've taken 1806 it's called a pole.

In a complex variable when you have a denominator that vanishes linearly we call it a pole. Things blow up. So this carrying amplitude has a pole off the real axis. And interpretation is correct. At this point, this function becomes infinite. And what is happening on the real line that A s is becoming large is just the remnant of that infinity over here that is affecting the value of this point. So in the complex plane you understand the function a little better. You see why it's becoming big and you can see also with a little [INAUDIBLE] why the phases shifting very fast because you have this point. And that's called the resonance. And this is the mathematically precise way of searching for resonances.

If you want to search for resonances what you should do is you have your formula for delta as a function of k. I mean, it's a complicated formula, but now try to solve the equation tan delta of this is equal to minus i because that's what guarantees that you have a pole that indeed it blows up at some value. That's where A s blows up which we see directly here-- it's this value. Alpha minus i beta, so alpha minus i beta is a pole of A s.

And therefore, you must be happening when tangent of delta is equal to minus i. So you have a very complicated formula maybe for tangent of delta. But set it equal to minus i and asked mathematically to solve it. And a number will come-- k a equal 2.73 minus 0.003. And you will know-- oh, that's a resonance, it's off the axis. And the real part is the value of alpha. And since this is beta the closer to the axis -- if you find more-- the more resonant it is. And by the time it's far from the axis, some people call it the resonance-- some people say, no that not the resonance. It's a matter of taste. But there are important things which are these poles. So I will not give you exercises on that, but you may want to try it if you want to have some entertainment with these things.

I want to say one more thing about this. And it's the reason why this viewpoint is interesting, as well. We already found that if we want to think of resonances more precisely. We can think of them as just an equation. You solve for the equation, so that it gives you the resonance. And this is the equation you must solve and you must admit complex k. But now you can say, look actually you have e is equal to h squared, k squared, over 2m. And we have real k's-- this is the physical scattering solutions, complex k's, also resonances.

How about imaginary k's? If k is equal to i kappa-- kappa belonging to the real numbers-- then the energy becomes minus h squared, kappa squared, over 2m and its less than zero and it could represent bound states. So you'll be then discovering solutions of real k representing your waves. Now mathematically, you are led to resonances understood as poles in the scattering amplitutde we did here. We see that k's in the imaginary axis would represent bound states. So the complex k-plane is very rich. It has room for your scattering solutions, it has room for your resonance, it even has room for your bound states. They're all there. That's why it's a valuable extension. I have now proven for you that bound states correspond to poles. It's a simple calculation, and that I would assign it to you with a little bit of guidance. And you will see that also for the case of bound states, you get a pole in the scattering amplitude, and that will complete the interpretation of that.

Now people go a little further, actually, and they invent poles in this part and they're called anti-bound states. And you'll say, what's that? If you have a bound state you match a solution to a pure decaying exponential for the [INAUDIBLE] region. In an anti-bound bound state you match your solution to a pure increasing exponential. A pure one. Does that have an interpretation? It actually does have interpretation. Some nuclear states are associated with anti-bound states.

So the mathematical description-- the rich complex plane is ready for you if you just do scattering amplitude k, resonances-- complex k. Normal bound states, imaginary k-- positive.

Anti-bounds is negative k. It's a nice start.