PROFESSOR: I want to just elucidate a little more what are the eigenstates here. So with angular momentum, we measure L squared and we measure Lz. So with spin, we'll measure spin squared and Sz. And Sz is interesting. It would be spin or angular momentum in the z direction.

So let's look at that, Sz, this is the operator, the measurable. It's this time nothing else than a simple matrix. It's not the momentum operator. It's not angular momentum operator with derivative. It's an angular momentum operator, but it seems to have come out of thin air.

But it hasn't. So here it is. Oh, and it's diagonal already. So the eigenstates are easily found. I have one state-- I don't know how I want to call it-- I'll call it 1, 0. It's one state. And that's an eigenstate of it. We'll call it, for simplicity, up. We'll see why.

Sz, acting on up, is equal to h bar over 2, 1, 0; 0, minus 1 acting on 1, 0. That's h bar over 2. And the matrix is at 1, 0. So it is an eigenstate because it's h bar over 2 up. The 1, 0 state again. So this thing, we call it up, because it has up component of the z angular momentum. So it's a spin up state.

What is the spin down state? It Would be 0, 1. It's a spin down. And Sz on the spin down, it's also an eigenstate, this time with minus h bar over 2, spin down.

And we call it spin 1/2 because of this 1/2. And you'd say, no, you just put that constant because you want it there. Not true, if I would have put a different constant here in defining this, I would not have gotten this without any constant, that it's how angular momentum works.

So if I use two-by-two matrices, I'm forced to get spin 1/2. You cannot get anything else. The 1/2 of the spin is already there. The component of angular momentum is h bar over 2. If you have a photon, it has spin 1. The components of angular momentum is plus h or minus h, if you have the two circularly polarized waves.

So this is actually interesting. But it begs for another question because we have a good intuition. And this is spin up along the z direction because it has a Sz component, eigenvalue h over 2. So the last question I want to ask is, how do I get a spin state to point in the x direction or in the y direction.

You see, the interpretation of this spin state is that it's a spin state that has the spin up in the z direction, because that's what you can measure, or spin down in the down direction of Sz. Can I get spin states that point along the x direction or y direction?

And here's where the problem seems to hit you and you say, I'm in trouble. I have this state spin up and spin down along z. And it's a two-dimensional vector space, because two-by-two matrices, and Sx, Sy, Sz is three dimensions. How am I going to get three dimensions out of two dimensions? You just have spin states along z, up and down.

Now the spin up and spin dow, moreover, are orthogonal states. These two are orthogonal states. You see, you do the inner product, transpose this, you get this, and times that. So they are orthogonal, unless you imagine this vector plus this vector is a full basis for the vector space, because the vector space is a, b. And now you see that this is a times up plus b times down.

So anything is a superposition of up and down. So how do I ever get something that points along x, or something that points along y? Well, let's try to see that. Well, consider Sx, you have an Sx operator, which is h bar over 2, 0, 1, 1, 0.

And then you can try to analyze this, but it's more entertaining to imagine other things, to say, look, if I've gotten this vector 1, 0, which is up, and 0, 1, which is down, I can try maybe a vector that has the up and the down. Maybe the up and the down is a vector that points nowhere. Who knows, whatever.

If I want to normalize it, I have to put a 1 over square root of 2. And now I know, it's 1 over square root of 2, up, plus down. That's what this vector is. But let's see what Sx does on it. Sx on 1 over square root of 2, 1, 1 is h bar over 2, 1 over square root of 2, 0, 1, 1, 0, on 1, 1. So h bar over 2, 1 over square root of 2.

And let's see, that gives 1, that gives me another 1. Oops, I got the same vector I started with. It an eigenstate. So this thing, this plus and down, superimposed, is an eigenstate of Sx. So this is actually a spin that points up, but in the x direction.

Whenever we don't put anything, we're talking about z. But this is the spin up in the x direction. And these appeared as the sum of a spin up and spin down in the z direction.

It may not be too surprising for you to imagine that if you put 1 over square root of 2, 1 minus 1, that vector is orthogonal to this one. Yes, you do the transpose. And this one is orthogonal. So this is 1 over square root of 2, up, minus, down. That is the down spin along x.

So the up and down spins along x come out like that. We form the linear combinations. So finally, you would say, well, I'm going to push my luck and try to get spins along the y direction.

But I now form those linear combinations. What else could I do? These linear combinations are there. And I've got already two things. And you say, well, that's fair, you're a two-dimensional vector space, so you're getting two things, spin states along x and spin states along z.

But actually, we didn't run out of things to try. We could try a state of the form 1 over square root of 2, something like this. We could try the state up. And then, we've put a plus, but now we could put a plus i, state down.

So this would be a state of the form 1, i. And what does it do? Well, let's see what it does with Sy. 1, i. And the Sy matrix is h bar over 2 minus i, i, 0, 0, 1, i. And there's 1 over square root of 2. So it's 1 over square root of 2, or h bar over 2, 1 over square root of 2.

And let's see what we get. Minus i times i is one 1. And the second one is i. We get the same state. Yes, it is an eigenstate. So with a plus i here, this is this spin up along the y direction.

And the spin down along the y direction would be up, minus i, down. This is orthogonal to that vector. It's 1 minus i. And it's the spin down in the y direction. You can calculate the eigenvalue, it's minus h bar over 2, and it's pointing down.

So your complex numbers play the crucial role. If you didn't have complex numbers, there was no way you could ever get a state that this pointing in all possible directions. And you also see, finally, that this thing has nothing to do with your usual wave functions, functions of x, theta, phi.

No, spin is an additional world with two degrees of freedom, an extra thing. It doesn't have a simple wave function. The spin wave functions are these two column vectors. But there is angular momentum in there, as you discovered here. There is a commutation relations of angular momentum, the units of angular momentum, the eigenvalues of angular momentum.

And this great thing is such a nice simple piece of mathematics. It has an enormous utility. It describes the spins of particles. So it's an introduction, in some sense, to what 805 is all about.

Spin systems are extremely important, practical applications. These things, because they have basically two states, are essentially qubits for a quantum computer. Within these systems, we understand, in the simplest way, entanglement, Bell inequalities, superposition, all kinds of very, very interesting phenomena. So it's a good place to stop.