PROFESSOR: I've put on the blackboard here the things we were doing last time. We began our study of stationary states that are not normalizable. These are scattering states. Momentum eigenstates were not normalizable, but now we have more interesting states that represent the solutions of the Schrodinger equation, that are stationary states with some energy e.

Because they are not normalizable, but we cannot directly interpret any of these solutions as the behavior of a particle. I kind of tell you a story, OK. So this is-- a particle is coming, colliding, doing something. These are not normalizable states. So part of what we're going to be trying to do today is connect to the picture of wave packets and see how this is used to really calculate what would happen if you send in a particle of potential.

Nevertheless, we wrote a solution that has roughly that interpretation, at least morally speaking. We think of a wave that is coming from the left, that's ae to the ikx. Now, in order to have a wave, you would have to have time dependents, and this is a stationary solution. And there is some time dependents. There is exponential of e to the minus iet over h bar, where e is the energy.

So this could be added here to produce the full stationary state psi of x and t. But we'll leave it understood-- it's a common phase factor for both the solutions of x less than 0, and x greater than 0, because the whole solution represents a single solution. It's not like a solution on the left and a solution on the right. It's a single solution over all of x 4 a psi that has some definite energy.

So we looked at the conditions of continuity of the wave function and continuity of the derivative of the wave function, at x equals 0. Those were two conditions. And they gave you this expression for the ratios of c over a and b over a. We could even imagine, since we can't normalize this, setting a equal to 1, And. Then calculating b and c from those numbers.

We have two case a k in a k bar. The k is relevant to the wave function for x less than 0. The k bar is relevant for x greater than 0. And they have to be different because this represents a DeBroglie wavelength, and the DeBroglie wavelength encodes the momentum of the particle, and the momentum of the particle that we imagine here classically is different here where this has this much kinetic energy, and in the region on the right, where the particle only has a much smaller kinetic energy. So that's represented by k bar.

And k bar, being the energy proportional to the energy minus v 0, while k squared has just the energy, it's smaller than k. So this is what we did. We essentially solved the problem, and this qualifies as a solution, but we still haven't learned anything very interesting from it. We have to understand more what's going on.

And one thing we can do is think of a particular limit. The limit, or the case, when e is equal to v 0, exactly equal to b 0, what happens? Well, k bar would be equal to 0. And if k bar is equal to 0, you're going to have just the constant c in here.

But if k bar is equal to 0, first b is equal to a, because then it's k over k, so b is equal to a. And c is equal to 2a. And the solution would become psi of x equals-- well, a is equal to b. So this is twice a cosine of kx, when a equal to b is a common factor, call it a. And this thing is the sum of two exponentials with opposite signs. That gives you the cosine.

And for c, you have 2a. And since k bar is equal to 0, well, it's just 2a. It's a number. This is unnormalizable, but even the original solution is unnormalizable, so we wouldn't worry too much about it. So how does that look for x over here? You have a cosine of kx, so it's going to be doing this to the left. That's for x less than 0.

And at this point, it goes like that. Just flat side effects, and here's 2a. so there's nothing wrong with the solution in this case. It's kind of a little strange that it becomes a constant, but perfectly OK.

What really helps you here is to find some conditions that express the conservation of probability. You see, you have a stationary state solution. Now, stationary states are funny states. They're not static states, completely static states. For example, if you have a loop, and you have a current that never changes in time. This is a stationary condition, even in electromagnetism.

So what we imagine here is that we're going to have some current, probability current, that is coming from the left, and some of it maybe bounces back, and some of it goes forward. But essentially, if you think of the barrier, whatever-- you look a little to the left of the barrier and a little to the right of the barrier.

Whatever is coming in, say, must be going out there, because probability cannot increase in this region. It would be like saying that the particle suddenly requires larger and larger probability to be in this portion of the graph. And that can't happen.

So probabilistic current gives you a way to quantify some of the things that are happening. So probability current plus j effects, which was h bar over m, imaginary part of psi star d psi dx.

So let's compute the probability current. Let's compute for x less than 0. What is the probability current j of x? We then call it the probability current on the left side. I would have to substitute the value of the wave function for x less than 0, which is the top line there, into this formula. And see what is the current.

In fact, I believe you've done that in an exercise some time ago. And as you can imagine, the current is proportional to the modulus of a squared, the length of a squared, the length of b squared also enters. And the funny thing is, between those two waves, one that is going to the right and one is going to the left, that that's very visible in the current. This is h bar k over m.

A squared minus b squared. It's a short computation, and it might be an OK and a good idea to do it again. It was done in the homework. And x greater than 0. J right of x would be equal to h bar k bar. Now that, you can almost do it in your head. This c into the ik bar x. Look what's happening.

From psi star you get a c star. From psi you get the c, so that's going to be a c squared. The face is going in a cancel between the one here and the one on psi, but the derivative will bring down an ik bar, and the imaginary part of that is just k bar. So the answer is this.

And these are the two currents. Now, if we are doing things correctly, the two currents should be the same. Whatever current exists to the left, say a positive current that is coming in, to the right must be the same. Another pleasant thing is that this current doesn't depend on the value of x, as x is less than 0. Nor of the value of x when x is greater than 0. And that's good for conservation. It would be pretty bad as well, if you look at two places for x less than 0, and you find that the current is not the same. So where is it accumulating? What's going on?

So the independence of these things-- from x, this is constant, and a constant is very important, because this constant should be the same. Now, whether or not about current conservation, it's encoded in Schrodinger's equation. And we solved Schrodinger's equation. That's how we got these relations between b, a and c So it better be that these two things are the same.

So Jay elsewhere-- jl, for example. I won't put off x, because it's just clear. It doesn't depend on x. 1 minus b over a squared a squared. I'm starting to manipulate the left one, and see if indeed the currents are the same. And now, I get h bar k over m. 1 minus b over a squared. I put the modulus squared, but so far everything is real in this-- k, k bar, b, a, c-- all real. Is that right? No complex numbers there. So I don't have to be that careful.

B over a squared is k minus k bar over k plus k bar squared a squared. And I gfit that here, maybe. H bar k over m. Now what? This is squared, so the si, squared passes here to the numerator minus the difference squared, that's going to be a 4k k bar over k plus k bar squared a squared.

And yes, this seems to be working quite well. Flip these k's-- these two k's, flip them around. So the answer for jl so far is h bar, now k bar, because I flipped that, m, and now I would have 4 k squared over this thing, which is 2k over k plus k bar squared of a squared. And this quantity, if I remember right, is just c squared, as you can see from there.

So indeed, this is j right, and it worked out. This is j right. So j left is equal to j right by current conservation. So that's nice. That's another way of getting insight into these coefficients. And that kind of thing that makes you feel that there's no chance you got this wrong. It all works well.