

PROFESSOR: Here is your potential. It's going to be a smooth, nice potential like that. V of x . x , x . And now, suppose you don't know anything about the energy eigenstates. Now, this potential will be assumed to be symmetric. So here is one thing you can do. You can exploit some things that you know about this potential.

And here's the wave function that we're going to try to plot. And we could say the following. Let's see. Whatever energy are here, for bound states, I'm going to eventually be in the forbidden region. So far on the right here, I will be in the forbidden region. And I must meet the wave function that looks like the forbidden region wave function. And the only possibility is something like that. You could say it's the [INAUDIBLE] of 1, but actually, if it's a [INAUDIBLE] then I could multiply by minus 1 and use this one conventionally. That wave function is always going to be like that over there.

On the other hand, very important-- the on the very left, how will the wave function look? Well it also has to decay, so it can decay like that, or it might be decaying like this. And in fact, you don't know until you figure out what's happening in the middle. It may be decaying like this or like that. I fix the sign here, so whatever this does it should either end up like that or end up like that. So these are the guidelines that you have to solve this. Should begin like that, and we'll see. And it should be either symmetric or anti-symmetric. This would be the case anti-symmetric, this would be the case symmetric.

OK. So let's draw one, two, three, four lines there. One, two, three, and four. And I don't know where the energies lie. I don't know what is the ground state energy. And I want to give you an insight into how you can figure out why you get this energy one decision when this happens. So let's plot the wave function for the first case. I don't know if I have a label, but let's assume this is E_0 , E_1 , E_2 , E_3 . Three energies [INAUDIBLE], and here's the one for e_0 .

OK. So I begin here, that's how it goes. And then I go through my Schrodinger equation, integrate it. You see? Numerical, you can always integrate the Schrodinger equation. And this should be always in this region, let me-- like this. And it's growing. And, oops, there should be turning points here. There should be turning points-- suppose this is-- I'm not going to get this so well, but it goes like this. And now this should be a turning point.

So I should change to the other type of curvature, curvature down. But what probably will

happen with E_0 is that it will switch and it will go like and start to curve, maybe. Well, if it looks like that, it must match to the development of the odd piece or the even piece. Now, it's never going to match with the odd one, so it might be with the even. And yes, it would match turning point here, but look what has happened here. You got the corner there.

You know, this was turning slowly, and this is starting to turn slowly, but here there is a discontinuity in the derivative. So this is not the solution. You try, but you fail. But that's-- right. Not every energy gives a solution. So they should have matched continuously and derivative continuously at that point, but it didn't have enough time to do that.

On the other hand, if we try the next one, maybe. The turning points will be here. Let's see what happens. Well, now the forbidden energies are over here, and now you have a turning point here that-- in here, the curvature is negative, the second derivative's curvature. And it's larger than it was here. Here it was small, here it's larger. So it's going to curve faster. Maybe if you get the E_1 right, it will curve enough so this flat here, in which case the other side will match nicely and you've got the solution. So you probably have to go little by little until this becomes flattened and, boom, you've got the solution. Energy eigenstate.

Let's go a little further. This graph continues there. Now I want to go to E_2 . How am I going to do that? I'm going to do it this way. So this was here, this was there. There is the vertical line here. And for E_2 , the turning points are even further out. And here is the wave function. And let's look at this thing that I have. Now, the turning point in this one corresponds to the E_2 turning point. This is E_1 .

And now this will go in here, we'll turn, and we will go curve and maybe do something like that. Because it's curving more and more, and faster. So by the time you reach here, this is no good, because this one will be symmetric. You know, you would have an anti-symmetric one that is no use. But now you don't have a solution, again. So as you increase the energy, this is starting to do this, and that is not quite so good. And then when you go to E_3 , you have a turning point over here. So maybe in this case it will go up here and it will start turning, and it will turn enough to just-- this dip go to the origin.

OK. You're saying no good either, because this is terrible, this continuous. But, ah, you were supposed to draw the other one as well. The old one is actually perfect for it. So this is dash, it doesn't exist, and this one matches here. So by the time the dip-- this is not a solution, but the dip goes down and down, and eventually goes to zero, it matches with this one. That's why I

said sometimes you don't know whether this matches with the one that comes from here or the one that comes from the bottom.

So there you go. This is an energy eigenstate again. It's odd and it has one node. And that gives you the intuition how, as you sort of come from the end and you reach the middle, you sometimes match things or sometimes don't match. And explains why you get energy quantization.

The other way in which you're going to gain intuition is with the so-called shooting method, which is the last thing I want to discuss for a minute. So the shooting method in differential equations is quite nice. Shooting method. Suppose you have a potential that this symmetric may be something like this-- it doesn't look very symmetric. It looks a little better now. And you want to find energy eigenstates.

You do the following. You say, well, the normalization of the energy eigenstates is not so important. Let's look for even states. Now, you can look for even or odd states if the potential is symmetric. Sometimes the potential will have a wall, in which case you have to require a symmetric potential. It's easy to solve, as well. But let's consider the case when the potential is symmetric and you look for even states.

So what you do is just, say, you pick an energy. Pick some energy E_0 . And then you put some boundary condition. You say that the wave function at x equals 0 is 1. And then you say that the derivative of the wave function at x equals zero is how much? Any suggestion? How much should it be?

You see, you have a second order differential equation. The second ψ is equal to E minus V . That's the Schrodinger equation. You need-- the boundary conditions are the value of ψ and the derivative at one point, and then the computer will integrate for you. Mathematica will do it. But you have to give me the derivative, so what should I put? A number there, 1, 2, 3? Is that an unknown? What should I pick?

We must put in the 0, because if you had a wave function whose derivative is not 0, and it's an even wave function, it would look like this. And there would be a discontinuity in ψ prime-- discontinuous. And that's not possible unless you have a hard wall or you have a delta function. So you must put this. And then you integrate numerically. Numerically.

And what will happen? Well, if you integrate numerically, the computer is just going to integrate

and see no problem. Basically, it's just going to do the interview. Ask the computer to calculate the wave function out to x equal 5, it will calculate it. So the problem is that-- you can see visually, if you pick some energy, the wave function will do like something, and then will start blowing up. And then you say, oh, that energy is no good because the wave function won't be normalizable.

And then you go back to the computer and change the energy a little bit, and then you will find, well, maybe this. Now it blows up in the other direction. No good either. But in some energy in between, as you change, there must be one in which it does the right thing, which is BK. Somewhere in between. And numerically, you change the value of the energy, you go-- in the shooting method, when you shoot it either goes up or down.

And you start working within those two numbers to restrict it until you get here. If you have a solution with five-digit accuracy, it will do this, this, this, and then blow up. If you have a solution with 10 digits after it, it will do this and go up to here and blow up again. You need 500-digits accuracy to get that wall. But it's a fun thing that you can do numerically and play with it. You can calculate five digits accuracy, ten digits accuracy within a matter of minutes.

It's very practical, and it's very nice, but one thing you have to do is clean up your equation before you start. You cannot have an equation in Mathematica with \hbar and m and all that. So you have to clean up the units, is the first step, and write it as an equation question without units. Your And this plots very nicely in Mathematica, and you will have lots of practice.