PROFESSOR: Important thing to do is to just try to understand one more thing. The creation and annihilation operators-- what do they do to those states? You see, a creation operator will I add one more a dagger, so somehow must change phi n into phi n plus 1.

A destruction operator with an a will kill one of these factors, and therefore it will give you a state with lower number of phi n minus 1. And we would like to know the precise relations. So look at this. Let's do with an A on phi n. And we know it should be roughly phi n minus 1. This is one destruction operator, but we can do it.

Look-- this is 1 over square root of n. A times a dagger to the n phi 0. A with a dagger to the n phi 0, we can replace by a commutator again. Commutator of a, a dagger to the n phi 0. This is 1 over square root of n factorial, and here we get a factor of n times a dagger to the n minus 1 phi 0.

You know, it's all a matter of those commutators we on the left blackboard. But this state-- by definition, we have n square root of n factorial. That's state, by definition, is phi n minus 1 times square root of n minus 1 factorial. See, by looking at this definition and saying, suppose I have n minus 1, n minus 1, this is phi n minus 1. So n minus 1 a daggers on phi 0 is n minus 1 factorial square root multiplied phi n minus 1.

And now we can simplify this-- square root of n factorial and square root of n minus 1 factorial gives you just a factor of square root of n that with this n here, this square root of n, phi n minus 1. Sop there we go-- here is the first relation. A is really a lowering operator. It gives you an eigenstate 1 less energy, but it gives it with a factor of square root of n, that if you care about normalizations, you better keep it.

That factor is there because the overall normalization of this equation was designed to make the states normalized. Similarly, we can do the other operation, which is what is a dagger acting on phi n. This would be 1 over square root of n factorial, but this time a dagger to the n plus 1 on phi n, phi 0.

Because you had already a dagger to the n, and you put one more a dagger. But this thing is equal to what? This is equal to square root of n plus 1 factorial times phi n plus 1. From the definition-- I hope you're not getting dizzy. Lots of factors here. But now you see that the n part of the factorial cancels, and you get that a hat dagger phi n is equal to square root of n plus 1 phi n plus 1.

OK let's do an application. Suppose somebody asks you to calculate example. The expectation value of the operator x on phi n, the expectation value of p on phi n. How much are they?

OK. This, of course, in conventional language, at first sight looks prohibitive. I would have to get those phi n [? in ?] some Hermit polynomial hn, for which I don't know the closed form expression. It's a very large polynomial, jumps 2 by 2, there is exponentials, I will have to do an integral.

That's something that we don't want to do. So how can we do it without doing integrals? Well, this one's-- actually, you can do without doing anything. You don't have to do integrals, you don't have to calculate. The answers are kind of obvious, if you think about it the right way. That's not the obvious part, to think about the right way.

But here it is. Look, what is this integral? This is the integral of x times phi and of x, those are real, quantity squared. And the phi n's are either even or odd, but the fight n squared are even. And x is odd. So this integral should be 0, and we shouldn't even bother. That's it.

Momentum. Expectation value of the momentum. All these are stationary states. Cannot have momentum. If it had momentum, here is the harmonic oscillator, here is the wave function. If it has momentum, half an hour later it's here. It's impossible. This thing cannot have momentum. This must be 0 as well.

OK. Now this one is something you actually proved in the first test-- the expectation value of the momentum operator on a bound state with a real wave function was 0. And you did it by integration-- but in fact you proved it in two ways, in momentum space, in coordinate space, is [? back ?] the same thing.

OK. So these ones were too easy. So let's try to see if we can find something more difficult to do. Well, actually, before doing that I will do them anyway with this notation. So what would I have here? I would have phi n x phi n. And I say, oh, I don't know how to do things with x. That's a terrible thing. I would have to do integrals.

But then you say, no. X-- I can write in terms of a and a daggers. And a and a daggers you know how to manipulate. So this is a formula we wrote last time, and it's that x is equal to square root of h over 2m omega, a plus a dagger.

So x is proportional to a plus a dagger. So here is a square root of h, 2m omega, phi n, a plus a dagger, on phi n. Now, this is 0, and why is that? Because this term is a acting on phi n. Well, we have it there-- is square root of n, phi n minus 1. And a dagger acting on phi n is square root of n plus 1, phi n plus 1.

But the overlap of phi n minus 1 with phi n is 0, because all these states with different energies are orthogonal. It's probably a property I should have written somewhere here. Which is-- not only they're well-normalized, but phi n phi m is delta nm.

If the numbers are different, it's zero. And you see this is something intuitively clear. If you wish, I'll just say here-- these are 0, and this is 0 because the numbers are different. If you have, for example, a phi 2 and phi 3, or let's do the phi 3 and a phi 2, then you have roughly a dagger, a dagger, a dagger, phi 0, a dagger, a dagger, phi 0. And then is equal to phi 0, three a's, and two a daggers. Correct?

And now you say, OK, this a is ready to kill what is on the right hand side. On the right side to it. But it can't because there are a daggers. But that a is going to kill at least one of the a daggers. So an a kills an a dagger. The second a will kill the only a dagger that is left. And now you have an a that is ready to go here, no obstacle whatsoever, and kills the phi 0, so this is zero.

So each time there are some different number of eight daggers on the left input and the right input, you get 0. If you have more a daggers on the right, then move them to the left, and now you will have more a's than a daggers and the same problem will happen. The only way to get something to work is they are the same.

But this of course is guaranteed by our older theorems that the-- eigenstates, if Hermitian operators with different eigenvalues are orthogonal. So this is nice to check things, but it's not something that you need to check.

All right. So now let's say you want to calculate the the uncertainty of x in phi n. Well, the uncertainty of x squared is the expectation value of x squared and phi n minus the expectation value of x on phi n. On this already we know is 0, but now we have a computation worth our tools.

Let's calculate the expectation value of x squared in phi n. And if you had to do it with Hermit polynomials, it's essentially a whole days work. Maybe a little less if you started using recursion relations and invent all kinds of things to do it. It's a nightmare, this calculation.

But look how we do it here. We say, all right, this is phi n x hat squared phi n. But x hat squared would be h bar over 2m omega, phi n times a plus a dagger time a plus a dagger phi n. Now I must decide what to do, and one possibility is to try to be clever and do all kinds of things.

Now, you could do several things here, and none is a lot better than the other. And all of them take little time. You have to develop a strategy here, but this is sufficiently doable that we can do it directly.

So what does it mean doing directly? Just multiply those operators. So you have phi n times a a plus a dagger a dagger plus a a dagger plus a dagger a. All that on phi n. I just multiplied, and now I try to think again. And I say oh, the first term is to annihilation operators acting on phi n.

The first is go give you phi n minus 1. Second is going to give me a phi n minus 2 by the time it acts. And a phi n minus 2 is orthogonal to a phi n. So this term cannot contribute. You know, this term has two more a's than this one. So as we just sort of illustrated, but it just doesn't match. These two terms acting on phi n would give you a phi n minus 2. And that's orthogonal.

So this term cannot do anything. Nor can this, because both raise. So this will end up as phi n plus two, for example, using that top property over there. Over there-- the box equation there. If you have two a daggers acting on phi n, you will end up with a phi n plus 2. So this term also doesn't contribute. And that's progress-- the calculation became half as difficult.

OK, that-- now we-- maybe it's a little more interesting. But again, you should you should refuse to do a [? long ?] computation. Whenever you're looking at those things, you have the temptation to calculate-- refuse that temptation. Look at things and let it become clear what's going on.

There are two terms here-- a dagger and a dagger a. That's not even a commutator, it's sort of like an anti-commutator. That's strange. But this, a dagger a, is familiar. That's n. The operator n. And we know the n eigenvalue, so this is going to be very easy. This is n hat.

The other one is not n hat, because it's in the wrong order. N hat has a dagger a. But this operator can be written as the commutator plus the thing in reverse order-- that equation we had on top-- ab is equal to ab commutator plus ba. So this is equal to a a dagger plus a dagger a. And this is 1. Plus another n hat.

So look-- when you have a and a dagger multiply, it's either n hat or it's 1 plus n hat. And Therefore x squared expectation value has become h bar over 2mw phi m, and this whole parenthesis is 1 plus 2 n hat phi n. And this is h bar over 2mw, phi n, and this is a number. Because phi in is an n hat eigenstate. So it's 1 plus two little n, phi n phi times 1 plus two [? little ?] n.

And here is our final answer-- expectation value of x squared is equal to h bar over and m omega, n plus 1/2 phi n. This is a fairly non-trivial computation. And that is, of course, because the expectation value of x is equal to zero, is the uncertainty or x squared. It grows, the state is bigger, as the quantum number n grows.

By a similar computation, you can calculate that you will do in the homework, the expectation value of b squared and phi n, and then you will see how much is delta x, delta p, on the [INAUDIBLE] on phi n. How much it is.