PROFESSOR: So we go back to the integral. We think of k. We'll write it as k naught plus k tilde. And then we have psi of x0 equal 1 over square root of 2pi e to the ik naught x-- that part goes out-- integral dk tilde phi of k naught plus k tilde e to the ik tilde x dk. OK.

So we're doing this integral. And now we're focusing on the integration near k naught, where the contribution is large. So we write k as k naught plus a little fluctuation. dk will be dk tilde. Wherever you see a k, you must put k naught plus k tilde. And that's it.

And why do we have to worry? Well, we basically have now this peak over here, k naught. And we're going to be integrating k tilde, which is the fluctuation, all over the width of this profile.

So the relevant region of integration for k tilde is the range from delta k over 2 to minus delta k over 2. So maybe I'll make this picture a little bigger.

Here is k naught. And here we're going to be going and integrate in this region. And since this is delta k, the relevant region of integration-- integration-- for k tilde is from minus delta k over 2 to delta k over 2. That's where it's going to range.

So all the integral has to be localized in the hump. Otherwise, you don't get any contribution. So the relevant region of integration for the only variable that is there is just that one.

Now as you vary this k tilde, you're going to vary the phase. And as the phase changes, well, there's some effect [? on ?] [? it. ?] But if x is equal to 0, the phase is stationary, because k tilde is going to very, but x is equal to 0. No phase is stationary.

And therefore, you will get a substantial answer. And that's what we know already. For x is going to 0 or x equal to 0, we're going to get a substantial answer.

But now think of the phase in general. So for any x that you choose, the phase will range over some value. So for any x different from 0, the face in the integral will range over minus delta k over 2x and to delta k over 2x.

You see, x is here. The phase is k tilde x. Whatever x is, since k tilde is going run in this range, the phase is going to run in that range multiplied by x.

So as you do the integral-- now think you're doing this integral. You have a nice, real, smooth function here. And now you have a running phase that you don't manage to make it stationary.

Because when x is different from 0, this is not going to be stationary. It's going to vary. But it's going to vary from this value to that value. So the total, as you integrate over that peak, your phase excursion is going to be delta k times x-- total phase excursion is delta k times x.

But then that tells you what can happen. As long as this total phase excursion is very small-- so if x is such that delta k times x is significantly less than 1-- or, in fact, I could say less than 1-- there will be a good contribution if x is such that-- then you will get a contribution.

And the reason is because the phase is not changing much. You are doing your integral, and the phase is not killing it. On the other hand, if delta k times x-- delta k times x is much bigger than 1, then as you range over the peak, the phase has done many, many cycles and is going to kill the integral.

So if k of x is greater than 1, the contribution goes to 0. So let's then just extract the final conclusion from this thing. So psi of x 0 will be sizable in an interval x belonging from minus x0 to x0.

So it's some value here minus x0 to x0. If, even for values as long as x0, this product is still about 1-- if for delta k times x0, roughly say of value 1, we have this. And therefore the uncertainty in x would be given by 2x0. So x0 or 2x0, this x0 is basically the uncertainty in x. And you would get that delta k times delta x is roughly equal to 1-- so delta k delta x roughly equal to 1.

So I'm dropping factors of 2. In principle here, I should push a 2. But the 2s, or 1s, or pi's at this moment are completely unreliable.

But we got to the end of this argument. We have a relation of uncertainties is equal to 1. And the thing that comes to mind immediately is, why didn't Fourier invent the uncertainty principle? Where did we use quantum mechanics here?

The answer is nowhere. We didn't use quantum mechanics. We found the relation between wave packets, known to Fourier, known to electrical engineers. The place where quantum mechanics comes about is when you realize that these waves in quantum mechanics, e to the ikx represent states with some values of momentum.

So while this is fine and it's a very important intuition, the step that you can follow with is-- it's interesting. And you say that, well, since p, the momentum, is equal to h bar k and that's quantum mechanical-- it involves h bar. It's the whole discussion about these waves of matter particles carrying momentum.

You can say-- you can multiply or take a delta here. And you would say, delta p is equal to h bar delta k. So multiplying this equation by an h bar, you would find that delta p, delta x is roughly h bar. And that's quantum mechanical.

Now we will make the definitions of delta p and delta x precise and rigorous with precise definitions. Then there is a precise result, which is very neat, which is that delta x times delta p is always greater than or equal than h bar over 2. So this is really exact. But for that, we need to define precisely what we mean by uncertainties, which we will do soon, but not today.

So I think it's probably a good idea to do an example, a simple example, to illustrate these relations. And here is one example. You have a phi of k of the form of a step that goes from delta k over 2 to minus delta k over 2, and height 1 over square root of delta k. That's phi of k. It's 0 otherwise-- 0 here, 0 there. Here is 0. Here is a function of k.

What do you think? Is this psi of x, the psi x corresponding to this phi of k-- is it going to be a real function or not? Anybody?

AUDIENCE: This equation [? is ?] [? true, ?] [? but-- ?]

PROFESSOR: Is it true or not?

AUDIENCE: I think it is.

PROFESSOR: OK. Yes, you're right. It is true. This phi of k is real. And whenever you have a value at some k, there is the same value at minus k. And therefore the star doesn't matter, because it's real.

So phi is completely real. So phi of k is equal to phi of minus k. And that should give you a real psi of x-- correct.

So some psi of x-- have to do the integral-- psi of x0 is 1 over square root of 2 pi minus delta k over 2 to delta k over 2. The function, which is 1 over delta k in here-- that's the whole function.

And the integral was supposed to be from minus infinity to infinity. But since the function only extends from minus delta k over 2 to plus delta k over 2, you restrict the integral to those values. So we've already got the phi of k and then e to the ikx dx.

Well, the constants go out-- 2 pi delta k. And we have the integral is an integral over x-- no, I'm sorry. It's an integral over k. What I'm writing here-- dk, of course. And that gives you e to the ikx over ix, evaluated between delta k over 2 and minus delta k over 2.

OK, a little simplification gives the final answer. It's delta k over 2pi sine of delta kx over 2 over delta kx over 2. So it's a sine of x over x type function.

It's a familiar looking curve. It goes like this. It has some value-- it goes down, up, down, up like that-- symmetric. And here is psi of x and 0. Here is 2 pi over delta k, and minus 2 pi over delta k here. Sine of x over x looks like that.

So this function already was defined with the delta k. And what is the delta x here? Well, the delta x is roughly 2 pi over delta k. No, it's-- you could say it's this much or half of that. I took [? it half ?] of that.

It doesn't matter. It's approximate that at any rate now. So delta x is this. And therefore the product delta x, delta k, delta x is about 2 pi.