Three-dimensional case. Now, in the future homework, you will be doing the equivalent of this calculation here with the Laplacians-- it's not complicated-- so that you will derive with the current is.

And the current must be a very similar formula as this one. And indeed, I'll just write it here. The current is h bar over m, the imaginary part of psi star. And instead of ddx, you expect the gradient of psi. That is the current for the probability in three dimensions.

And the analog of this equation, d rho dt plus dj dx equals 0, is d rho dt plus divergence of j is equal to 0. That is current conservation. Perhaps you do remember that from your study of electromagnetism. That's how Maxwell discovered the displacement current when he tried to figure out how everything was compatible with current conservation. Anyway, that argument I'll do in a second so that it will become clearer.

So one last thing here-- it's something also-- you can check the units here of j is 1 over l squared times 1 over t, so probability per unit area and unit time.

So what did we have? We were doing the integral of the derivative of the integral given by n. It was over here, dn dt. We worked hard on it. And dn dt was the integral of d rho dt. So it was the integral of d rho dt dx.

But we showed now that d rho dt is minus dj dx. So here you have integral from minus infinity to infinity dx of dj dx. And therefore, this is-- I should have a minus sign, because it was minus dj dx. This is minus the current of x equals infinity times p minus the current at x equals minus infinity nt.

And as we more or less hinted before, since the current is equal to h over 2im psi star [INAUDIBLE] psi dx minus psi [INAUDIBLE] psi star dx, as you go to plus infinity or minus infinity, these things go to 0 given the boundary conditions that we put. Because psi or psi star go to 0 to infinity, and the derivates are bound at the infinity.

So this is 0, dn dt 0. All is good. And two things happened. In the way of doing this, we realized that the computation we have done pretty much established that this is equal to that, because dn dt is the difference of these two integrals, and we showed it's 0. So this is true.

And therefore, we suspect h is a Hermitian operator. And the thing that we should do in order to make sure it is is put two different functions here, not two equal functions. This worked for two equal function, but for two different functions, and check it as well. And we'll leave it as an exercise. It's a good exercise.

So this shows the consistency. But we discover two important ideas-- one, the existence of a current for probability, and two, h is a Hermitian operator.

So last thing is to explain the analogy with current conservation. I think this should help as well. So the interpretation that we'll have is the same as we have in electromagnetism. And there's a complete analogy for everything here. So not for the wave function, but for all these charge densities and current densities.

So we have electromagnetism and quantum mechanics. We have rho. Here is the charge density. And here is the probability density. If you have a total charge q in a volume, here is the probability to find the particle in a volume.

There is a j in Maxwell's equations as well, and that's a current density. Amber's law has that current. It generates the curl of b. And here is a probability current density. So that's the table.

So what I want to make sure is that you understand why these equations, like this or that, are more powerful than just showing that dn dt is 0. They imply a local conservation of probability. You see, there has to be physics of this thing. So the total probability must be 1.

But suppose you have the probability distributed over space. There must be some relation between the way the probabilities are varying at one point and varying in other points so that everything is consistent. And those are these differential relations that say that whenever you see a probability density change anywhere, it's because there is some current.

And that makes sense. If you see the charge density in some point in space changing, it's because there must be a current. So thanks to the current, you can learn how to interpret the probability much more physically. Because if you ask what is the probability that the particle is from this distance to that distance, you can look at what the currents are at the edges and see whether that probability is increasing or decreasing.

So let's see that. Suppose you have a volume, and define the charge inside the volume. Then you say OK, does this charge change in time? Sure, it could. So dq dt is equal to integral d rho dt d cube x over the volume.

But d rho dt, by the current conservation equation-- that's the equation we're trying to make sure your intuition is clear about-- this is equal to minus the integral of j-- no, of divergence of j d cube x over the volume.

OK. But then Gauss's law. Gauss's Lot tells you that you can relate this divergence to a surface integral. dq dt is therefore minus the surface integral, the area of the current times that. So I'll write it as minus jda, the flux of the current, over the surface that bounds-- this is the volume, and there's the surface bounding it.

So by the divergence theorem, it becomes this. And this is how you understand current conservation. You say, look, charge is never created or destroyed. So if you see the charge inside the volume changing, it's because there's some current escaping through the surface.

So that's the physical interpretation of that differential equation, of that d rho dt plus divergence of j is equal to 0. This is current conservation. And many people look at this equation and say, what? Current conservation? I don't see anything.

But when you look at this equation, you say, oh, yes. The charge changes only because it escapes the volume, not created nor destroyed. So the same thing happens for the probability.

Now, let me close up with this statement in one dimension, which is the one you care, at this moment. And on the line, you would have points a and b. And you would say the probability to be within a and b is the integral from a to b dx of rho. That's your probability. That's the integral of psi squared from a to b.

Now, what is the time derivative of it? dp ab dt would be integral from a to b dx of d rho dt. But again, for that case, d rho dt is minus dj dx. So this is minus dx dj dx between b and a.

And what is that? Well, you get the j at the boundary. So this is minus j at x equals bt minus j at x equal a, t. So simplifying it, you get dp ab dt is equal to minus j at x equals bt plus j of a, t.

Let's see if that makes sense. You have been looking for the particle and decided to look at this range from a to b. That's the probability to find it there. We learned already that the total probability to find it anywhere is going to be 1, and that's going to be conserved, and it's going to be no problems.

But now let's just ask given what happens to this probability in time. Well, it could change, because the wave function could go up and down. Maybe the wave function was big here and a little later is small so there's less probability to find it here.

But now you have another physical variable to help you understand it, and that's the current. That formula we found there for j of x and t in the upper blackboard box formula is a current that can be computed. And here you see if the probability to find the particle in this region changes, it's because some current must be escaping from the edges.

And let's see if the formula gives it right. Well, we're assuming quantities are positive if they have plus components in the direction of x. So this current is the current component in the x direction. And it should not be lost-- maybe I didn't quite say it-- that if you are dealing with a divergence of j, this is dj x dx plus dj y dy plus dj z dz. And in the case of one dimension, you will have those, and you get this equation. So it's certainly the reduction.

But here you see indeed, if the currents are positive-- if the current at b is positive, there is a current going out. So that tends to reduce the probability. That's why the sign came out with a minus.

On the other hand, if there is a current in a, that tends to send in probability, and that's why it increases it here. So the difference between these two currents determines whether the probability here increases or decreases.