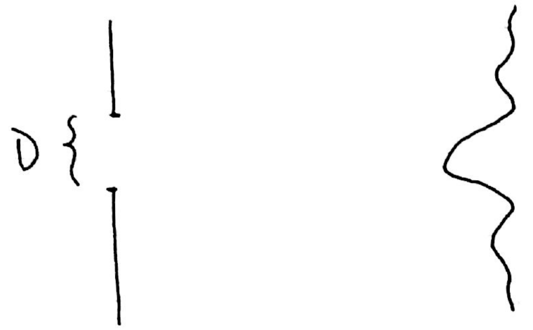
We have learned single slit diffraction

$$I = I_0 \left(\frac{\sin\beta}{\beta} \right)^2 \qquad \beta = \frac{\pi D \sin\theta}{\lambda}$$



This means that a laser pointer is not merely producing a pencil beam.

Suppose $\lambda = 500\ nm = 5 \times 10^{-7}$

$D = 1\ mm = 1 \times 10^{-3}$

Opening angle :

$$\theta \approx \overbrace{1.22}^{\text{circular source}} \frac{\lambda}{D} = 6 \times 10^{-4}$$

If we shoot a laser to moon : $L = 4 \times 10^{8}\ m$

Radius of the principle maxima : 240 km !!
$L \cdot \theta$

Until now:

We have seen waves of matter

waves of vector field

(EM waves)

→ They provided a completely adequate description of nature

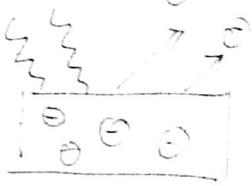
The light we know so far:

Like waves.

But since 20th century:

It was found that light did indeed behave like a particle sometimes.

ie Photoelectric Effects $E = h\nu$

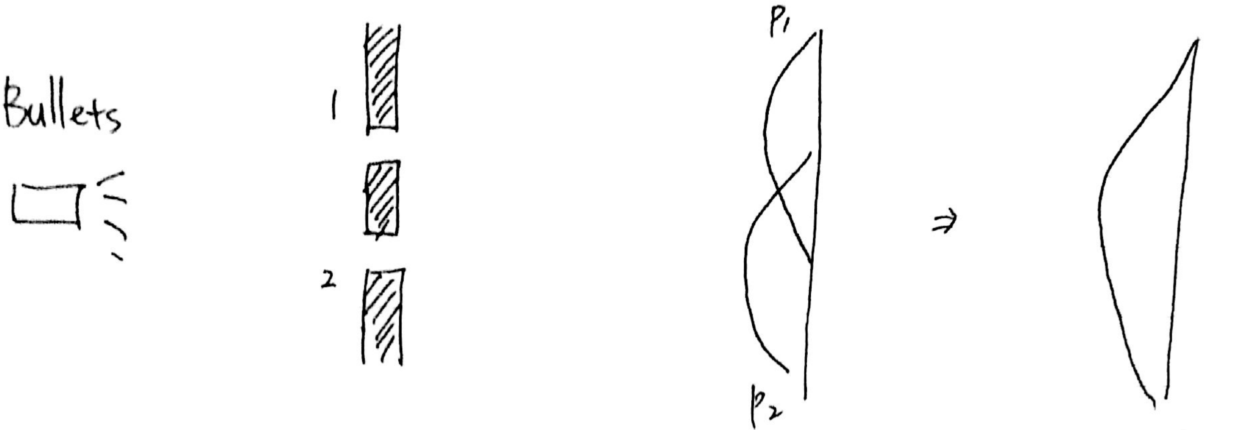


Electrons are elementary particles, but they do sometimes behave like waves!

De Broglie's matter wave: $p = \hbar k$

Example: Electron interference experiment.

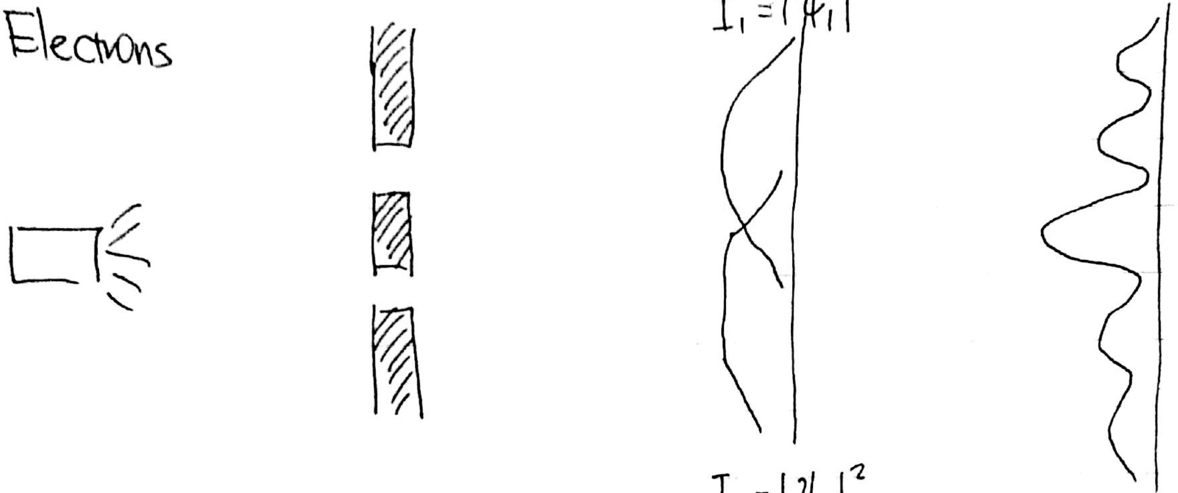
Bullets



$P_{12} = P_1 + P_2$

〈Exp1〉

Electrons



$I_1 = |\psi_1|^2$

$I_2 = |\psi_2|^2$

$$I_{12} = |\psi_1 + \psi_2|^2$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

Electrons :

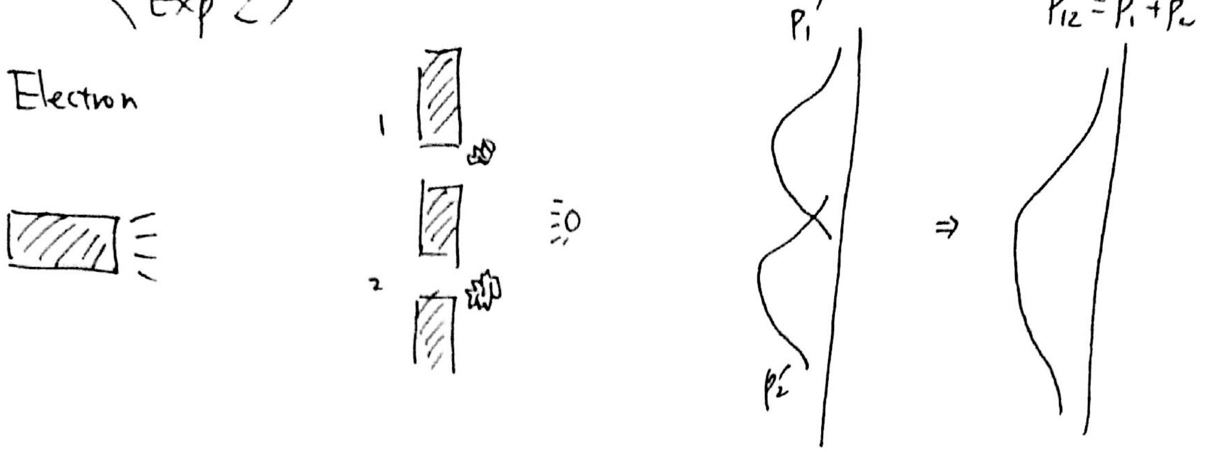
(1) Arrives like a particle. Produce a hit in the detector

(2) Interfers with itself like a wave.

Electron is like neither of them in reality!

How about we add a light ?

<Exp 2>



if electron passes through 1
→ see scattered light from 1.

→ We know which hole the electron actually pass through !

→ then the $\overset{\text{observed}}{}$ distribution become like bullets !?

No interference. (Electron is "disturbed" !)

If we lower the intensity of the light , what will happen ?

We found that the light is a lot of photons.

Lower the intensity → fewer photons

→ Sometimes it is hitting the electron
Sometimes not !

⇒ the distribution observed is a mixture of exp 1 and exp 2!

Wait! Can we still lower the energy further?

Photon energy

$$E = h\nu = \frac{hc}{\lambda}$$

Change the wave length!

We found that if the wave length of light is larger than the distance between slits

→ Interference pattern appears

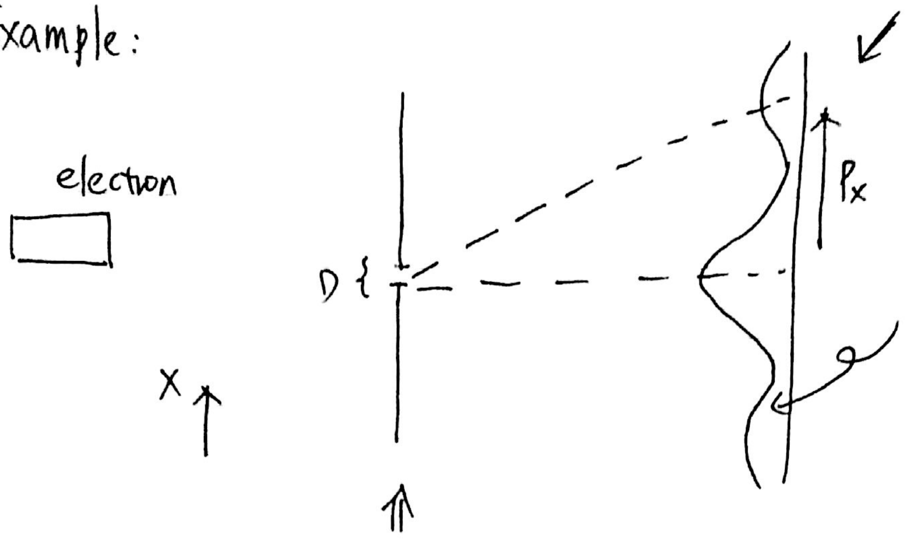
∵ We are not sure anymore which slit the electron actually pass through!

It is not possible yet to tell the position of the electron at the same time not disturbing it ⇒

Heisenburg's Uncertainty Principle!

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

Example:

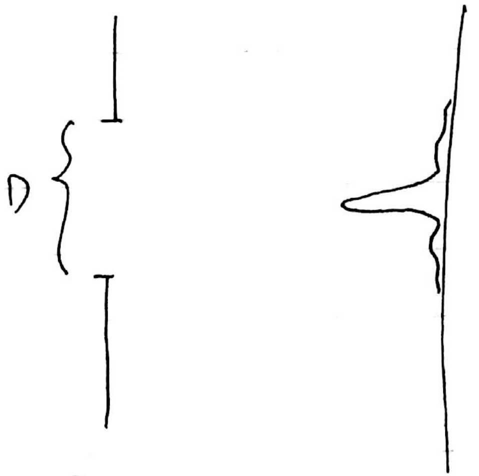


we are not sure about the momentum in the x direction!

wide distribution

We are very sure about the x position of the electron

On the other hand



$$C(k_x) \propto \int_{-\frac{D}{2}}^{\frac{D}{2}} dx \; e^{-ik_x x}$$

$D \to \infty \quad \Rightarrow \quad C(k_x) \to \delta$ function

Nobody was able to find a work around yet. If you can, QM has to be discarded

From this experiment:

The position of the electron is

described by a "wave function" ψ

The probability to find the electron:

$$\mathbb{P} \propto |\psi|^2$$

This is one of the most crazy result in Physics.

Quantum Mechanics tells us:

We can only predict the odd !!!

We predict the exact evolution of wave function

which gives the probability distributions.

But NOT THE OUTCOME !!

We believe now it is impossible to predict exactly what would happen in a given situation.

Hidden varible ?

The electron may already made up its mind which hole to pass through

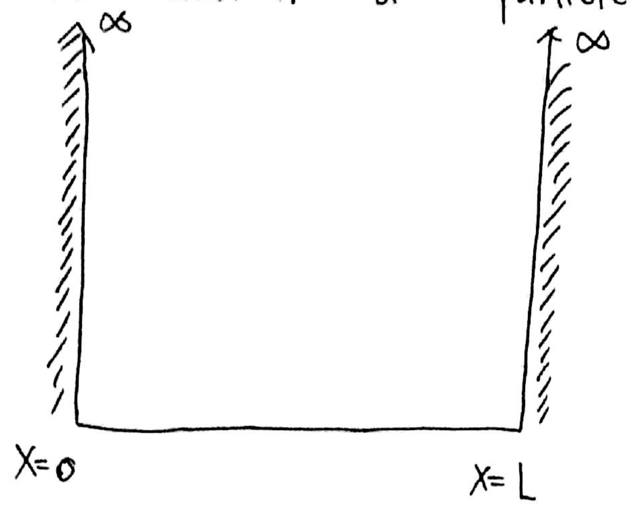
→ Not possible, it should not depend on what we do.

Now we have a brand new kind of wave : Probability wave !

Probability $\propto |\psi|^2$

Break.

Let's consider a particle in a box:



The wall (potential) is very very high. (∞).

What is the wave function $\psi(x)$ of the "normal modes"?

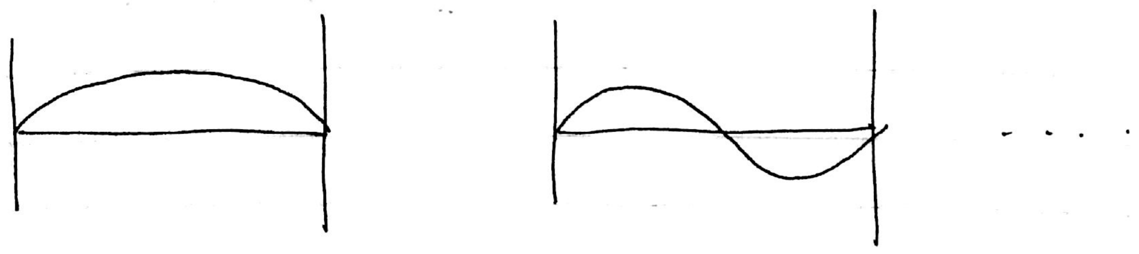
Boundary conditions:

$$\psi(0)=0 \quad , \quad \psi(L)=0$$

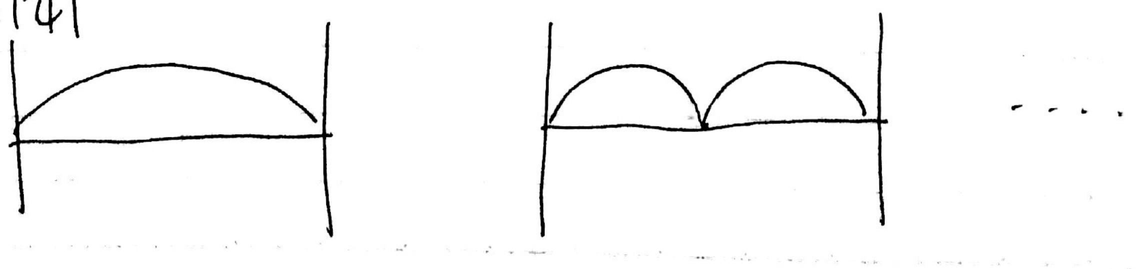
$$\Rightarrow \psi_m(x) = A_m \sin(K_m x) \qquad K_m = \frac{m\pi}{L}$$

$$\psi_m(x,t) = A_m \sin(k_m x)\, e^{-i\omega_m t} \qquad m = 1, 2, 3, \ldots$$

ψ



$P = |\psi|^2$



What is still missing : "Wave equation"

It turns out that the wave equation is

Schrödinger's Equation :

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x,t) \right] \psi(x,t)$$

Feynman (Lecture on Physics)

" Schrödinger's Equation from where ?

It is not possible to derive it from

anything you know. It came out of

the mind of Schrödinger !! "

Plug $\psi_m(x,t)$ into the equation : ($V(x,t)=0$ in the box)

$$\hbar \omega_m \psi_m = \frac{\hbar^2 k_m^2}{2m} \psi_m$$

Dispersion relation :

$$\omega = \frac{\hbar k^2}{2m}$$

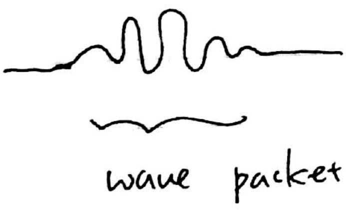
De Broglie's matter wave:

momentum: $p = \hbar k$

$$\Rightarrow \quad v_g = \frac{d\omega}{dk} = \frac{\hbar k}{m} = \frac{\overset{\text{momentum}}{p}}{m} \quad !!!$$

Group velocity ⇒ classical velocity !

A "particle": Superposition of many many waves with different wave lengths



8.03: this packet is traveling at group velocity!

This is not the end of waves and vibrations but just the begining !!!

8.03SC Physics III: Vibrations and Waves
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