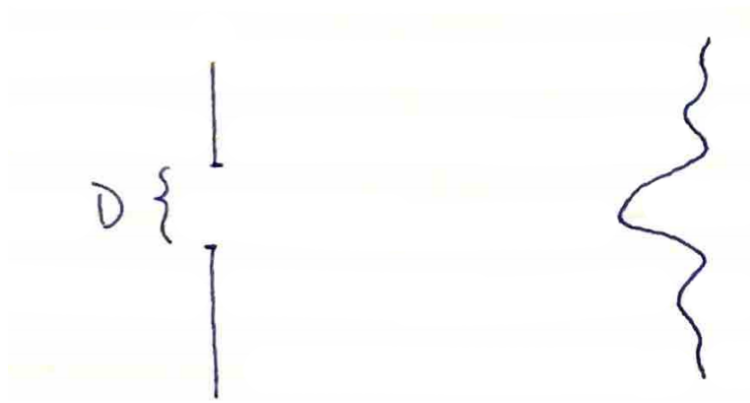


8.03 Lecture 23

We have learned single slit diffraction.

$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \quad \beta = \frac{\pi D \sin \theta}{\lambda}$$



This means that a laser pointer is not merely producing a pencil beam.

Suppose $\lambda = 500 \text{ nm} = 5 \times 10^{-7} \text{ m}$ and $D = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$.

Opening angle:

$$\theta \approx 1.22 \frac{\lambda}{D} = 6 \times 10^{-4}$$

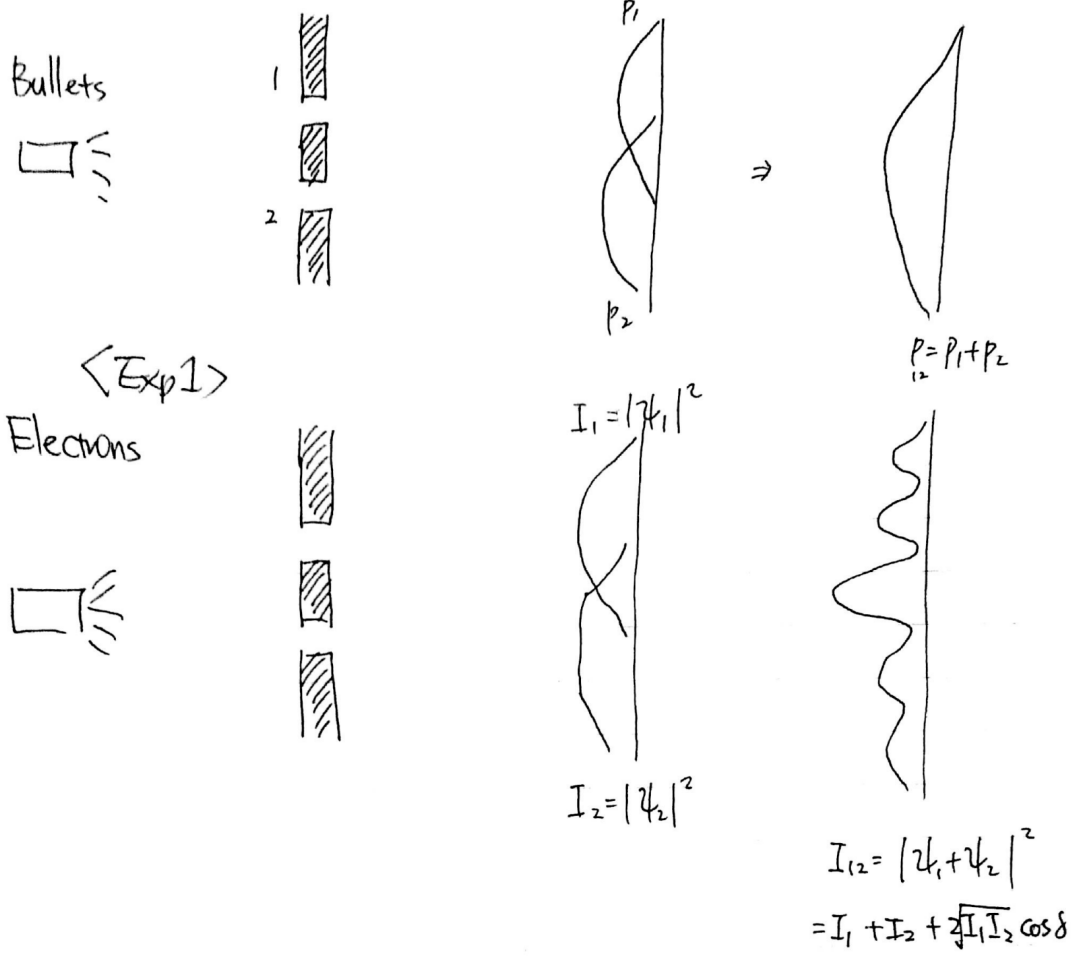
If we shoot a laser to moon: $L = 4 \times 10^8 \text{ m}$ the radius of the principle maxima is 240 km!!

Until now we have seen waves of matter, waves of vector field (EM waves). They provided a completely adequate description of nature.

The light we know so far is like waves. But since the 20th century it was found that light did indeed behave like a particle sometimes (i.e the photoelectric effect).

Electrons are elementary particles, but they do sometimes behave like waves (e.g. De Broglie's matter wave)!

Example: electron interference experiment.

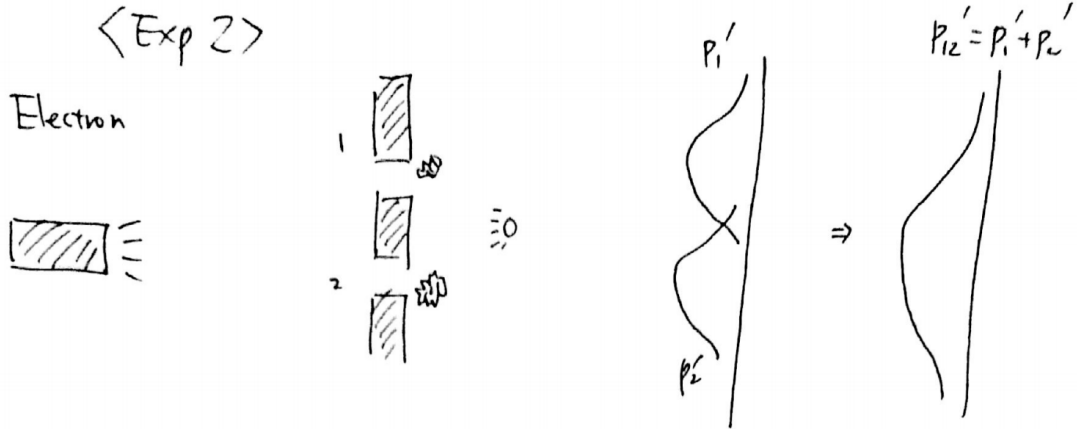


Electrons:

1. Arrive like a particle. Produce a hit in the detector.
2. Interferes with itself like a wave.

Electron is like neither of them in reality!

How about we add a light?



If electron passes through 1 → see scattered light from 1.

We know which hole the electron actually passes through!

Then the observed distribution becomes like bullets?!

No interference. (Electron is “disturbed”!)

If we lower the intensity of the light, what will happen? We found that the light is a lot of photons.

Lower the intensity → fewer photons. Sometimes it is hitting the electron, sometimes not!

Then the distribution observed is a mixture of experiment 1 and experiment 2!

Wait! Can we still lower the energy further?

$$E = h\nu = \frac{hc}{\lambda}$$

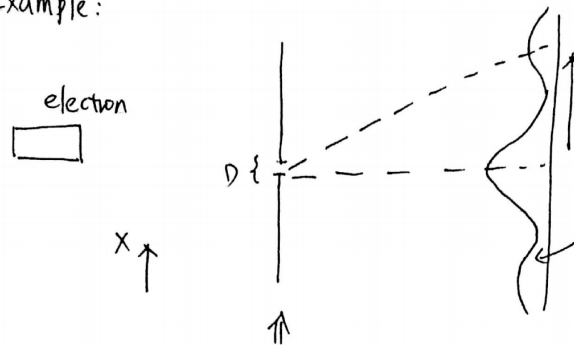
Change the wavelength! We found that if the wave length of light is larger than the distance between slits, then an interference pattern appears. ∴ We are not sure anymore which slit the electron actually passed through!

It is not possible yet to tell the position of the electron at the same time without disturbing it. ⇒

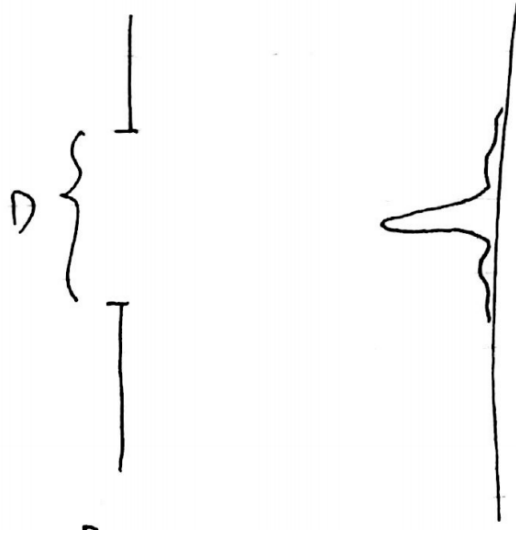
Heisenberg’s Uncertainty Principle:

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

Example:



We are not sure about the momentum in the x direction. We are very sure about the x position of the electron. Conversely:



$$C(k_x) \propto \int_{-D/2}^{D/2} e^{-ik_x x} dx$$

$$D \rightarrow \infty \Rightarrow C(k_x) \rightarrow \delta \text{ function}$$

Nobody was able to find a work around yet. If you can, quantum mechanics has to be discarded.

From this experiment: The position of the electron is described by a “wave function,” ψ . The probability to find the electron: $P \propto |\psi|^2$. This is one of the most crazy results in physics.

Quantum Mechanics tells us: We can only predict the odd!!! We predict the exact evolution of the wave function which gives us the probability distribution, BUT NOT THE OUTCOME!!

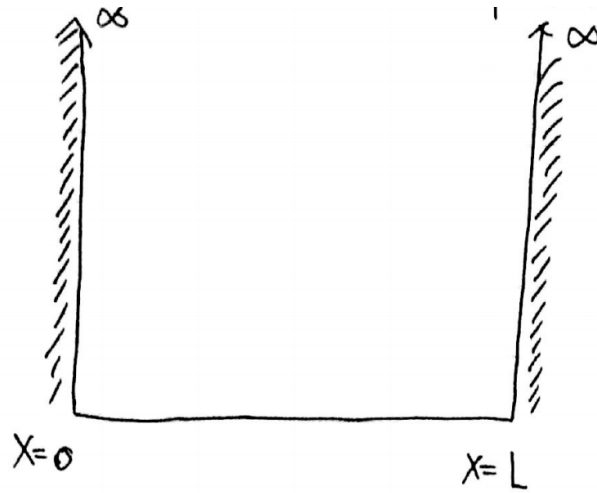
We believe now it is impossible to predict exactly what would happen in a given situation.

Hidden variable?

The electron may already make up its mind about which hole to pass through. Not possible, it should not depend on what we do.

Now we have a brand new kind of wave: probability wave! Probability $\propto |\psi|^2$

Let's consider a particle in a box:

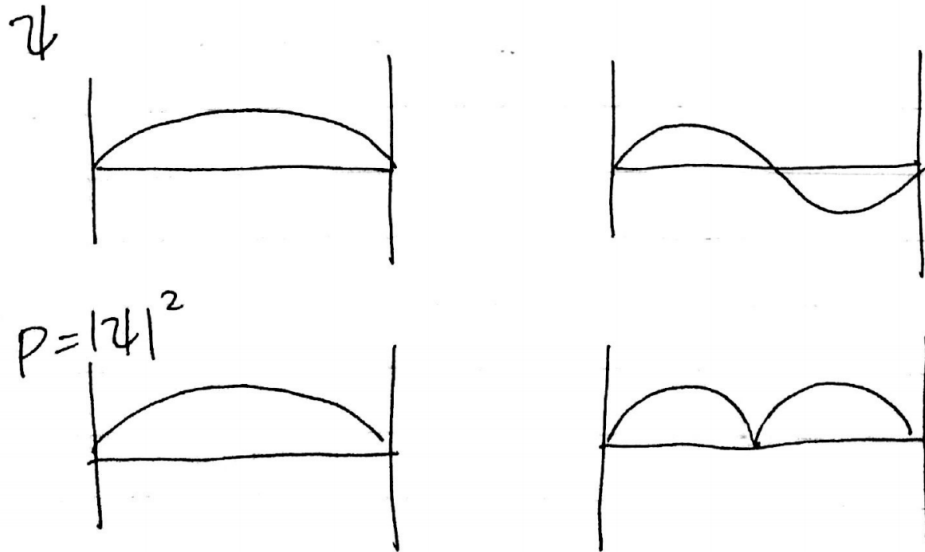


The wall (potential) is very very high, ∞ . What is the wave function $\psi(x)$ of the "normal modes"?
Boundary Conditions:

$$\psi(0) = 0 \quad , \quad \psi(L) = 0$$

$$\Rightarrow \psi_m(x) = A_m \sin(k_m x) \quad k_m = \frac{m\pi}{L}$$

$$\psi_m(x, t) = A_m \sin(k_m x) e^{-i\omega_m t} \quad m = 1, 2, 3, \dots$$



What is still missing: "Wave equation":

It turns out that the wave equation is Schrödinger's Equation:

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) \right] \psi(x, t)$$

From Feynman (Lectures on Physics): "Schrödinger's Equation from where? It is not possible to derive it from anything you know. It came out of the mind of Schrödinger!"

Plug in $\psi_m(x, t)$ into the equation (note $V(x, t) = 0$ in the box).

$$\hbar\omega_m \psi_m = \frac{\hbar^2 k_m^2}{2m} \psi_m$$

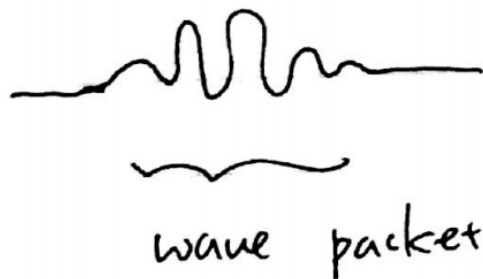
and the dispersion relation:

$$\omega = \frac{\hbar k^2}{2m}$$

De Broglie's matter wave: Momentum: $p = \hbar k$

$$\Rightarrow v_g = \frac{d\omega}{dk} = \frac{\hbar k}{m} = \frac{p}{m}!!!$$

Group velocity is the classical velocity! A "particle": superposition of many waves with different wave lengths.



In 8.03 this packet is traveling at group velocity!

This is not the end of waves and vibrations but just the beginning!!!

MIT OpenCourseWare
<https://ocw.mit.edu>

8.03SC Physics III: Vibrations and Waves
Fall 2016

For information about citing these materials or our Terms of Use, visit: <https://ocw.mit.edu/terms>.