PROFESSOR: Welcome back, everybody to 8.03. Very happy to see you again. So as you can see from the slides, we will continue the discussion from last time. We were talking about interference phenomena, which involve two or multiple point light source.

And they actually interact with each other and produce interesting phenomenon, which we see with laser, with water ripples, and also we discussed how to design a phased radar together.

And one thing which we learned is that if you, for example, have two slit interference, OK. And if you look at the intensity of the resulting interference pattern as a function of angle, you will see that there are peaks, periodic peaks as a function of angle.

And we also know how to calculate where would be the principal maxima, what would be the minima, which will have destructive interference between the two point light source. OK? So as usual, we go from one electromagnetic wave to two electromagnetic waves and two unelectromagnetic waves.

And today we are going to do infinite number of electromagnetic waves and they are going to interact with each other or superpose an infinite number of electromagnetic waves all together. And that brings us to that discussion of diffraction. OK? So what are we going to talk about today is, for example, a point light source, a laser pointer.

And what would the image of a laser pointer look like? When these lasers pass through a single slit or just the laser itself. The laser beam itself, what will happen to this laser beam?

And also we will make some comments on the Star Trek, for example. Right? They have this super weapon which they shoot enemy with this laser beam. And we'll see how practical that is by the end of this course. And the third thing is that it's related to resolution. So we are going to design a phone, screen of your mobile phone together to see what is actually practical, what is actually not practical.

If Yen-Jie is opening a new company to develop jPhone, what should be the requirement for the screen, for example? Which, I'm not going to do it.

So this is actually what we are going to discuss today. So we are interested in a situation where you have plane waves, and those plane waves are approaching from the left-hand side of the screen toward a single slit. So basically, the setup is like this.

So you have those wavefront basically is traveling to the right-hand side, the plane waves. And on the wall, there's a slit or hole, which is actually a opening, and the waves can actually penetrate through this hole.

The width of this hole is denoted by, or essentially given to you, which is actually D. And we were wondering what is going to happen to-- what are we going to observe on the screen, which is actually pretty far away from the wall. And this screen is actually used to observe the pattern of the interference pattern of the electromagnetic wave passing this hole.

So, as we discussed last time, due to Huygens' principle, every point is actually like a point-like source of spherical waves. So, as you can see now, we actually consider the size of our slit. Therefore, there must be a lot of point-like source inside, in this slit. When this wavefront actually pass through this wall, there should be infinite number of point-like source.

And all of them, due to Huygens' principle, is going to be like point-like source of spherical waves. And they are all emitting from all those possible location, and that they are overlapping each other and they have constructive or destructive interference with each other. So that is actually what is happening with this single-slit experiment.

And we call that diffraction. So you may be wondering, why do I call it diffraction? Why not interference? Because it's basically the same phenomenon, right? I think it's just a matter of wording.

Feynman actually commented on this, and he said that, nobody was able to define the difference between diffraction and interference in a satisfactory way. Which is actually true. So it's just a matter of wording. So we are looking at exactly the same phenomena when we actually discuss this experiment.

So what I am going to do today is now to introduce to you the way we can deal with this. I'm sure you have seen this experiment before, maybe in 8.02 or in high school days. On the other hand, what we are going to do today is to really make use of the mathematics which we have learned from 18.03 or from the previous lectures to attack this problem.

So what is actually the mathematics I am going to use today? So the mathematics which I would like to use to attack this problem is to use a two-dimensional Fourier transform. I think by now you should not be afraid of Fourier transform any more. It should be pretty natural. It's just integration, and you evaluate, and then you are going to get the corresponding number whatsoever.

But the cool thing is that 18.03 give them physical meaning of those numbers, and I'm going to talk about that. So what is actually the Fourier transform I'm going to use? So I am going to evaluate C, which is a function of kx and ky. And what is actually this C function? This C function is equal to 1 over 4 pi squared, which I really don't care too much. It's just a constant.

And I do the integration from minus infinity to infinity for dx, and I do a integration from minus infinity to infinity dy, a small letter scale, dy. And I have a f function, which I will introduce you what the f function mean, what does the f function actually represent. And exponential minus i, k is the vector which is actually telling you the direction of the propagation of the spherical wave, times r, which is actually a function of x and y.

And this is actually the kind of integration which we will employ in order to attack the problem we are interested in this lecture. So, what does this integration mean? So we have basically some kind of this two-dimensional Fourier transform.

The f function is actually telling you the shape of the source. So basically, this is actually telling you about the shape of the source. As we discussed before, the shape of the source, every point on this shape is a source of spherical wave, by Huygens' principle. So that is actually telling you where should I do the integration.

This one, exponential i k dot r, what is that? This is actually telling you about the spherical wave. So remember, we were doing two-slit interference before, and we have actually two exponential function, if you remember from last lecture.

So now, this is actually put there because each source you are going to get exponential function, which is actually presenting the propagation of the electromagnetic wave. You can say that, oh, wait, wait, wait. The omega t disappeared, right? There's no omega t here, right?

But I don't really care because everybody is actually oscillating at the same frequency, the same phase. Therefore, I factorize out. After I have done all the calculation, I can multiply the whole thing by cosine omega t, and probably some phi. Then that is actually modulating and oscillating up and down as the plane wave, as you approach the wall. So, therefore, I actually already factorize it out.

So this is actually telling you about the electric field. And what is actually here? This is actually the unit area you are performing this integration. And you can actually do integration over the full universe.

So you have a plane which actually extend to the whole full universe. But what is actually really contributing is defined by this f function, which is actually the shape of the source. And some normalization factor, which I don't really care too much.

So this looks really fancy, but is actually not that fancy. And what product you are getting here is our C function, is that C is actually a function of kx and ky. What is kx and ky? It's actually telling you the direction of propagation. The k vector is actually telling you the direction of the propagation.

If you evaluate C with a specific given kx and ky, basically you are evaluating the total electric field going some direction, which is actually defined by kx and ky.

So the big picture is the following. So basically you have some source. It can look like this in the xy plane. This is x and y plane. All those things, all those points, all those little areas inside this hole is spherical wave source.

And the f function actually define the shape of this hole. And this integration is actually integrating over all those little areas. And then calculate the contribution from each small area, sum them together. Then, finally, you are getting something which is actually a function of kx and ky.

What is kx and ky? It's actually giving you the direction of propagation from this point-like source to observer P. And this C function is actually proportional to the total electric field.

So you can see that, hah, we have learned this Fourier transform from the math department, and we give life to this function. Now actually we understand what we are doing now. We are actually really summing all over the, summing over the available point-like source.

And add all the contribution of the electromagnetic wave together. Then what we are getting, the C function is actually proportional to the total electric field. So that is actually the big picture. Any questions so far? I hope you can actually understand what we are doing.

So now what I am going to do is to really use this formula and attack the problem which we are actually trying to understand, the single-slit problem. So suppose I have a single slit which looks like this. I'm zooming in this thing maybe 100 times, 1,000 times. And this is actually a wall. It's very, very long.

I would like to define first my coordinate system. The x direction, as I actually drew from there, is actually pointing upward. The y direction is actually parallel to the wall. And the z direction is actually going to where the screen which I am trying to display the outcome of this experiment. And the distance between these two walls is actually D, which is actually given there.

So I would like to actually understand what is going to happen when the plane waves pass through with this single slit. Therefore, before I calculate the C function proportional to the total electric field, what I really need is a functional form, f, which describe this single slit.

And just to make sure that everybody is on the same page, this wall is actually infinitely long, from minus infinity in y to positive infinity y. So it's actually a super long wall. And these two edge is actually-- the distance between the edges is actually D.

So what would be the f function which describes the shape of the light source? f function is a function of x and y. And I define a f function, and I give you this function to describe the experimental setup.

So the f function can be either 1, which actually shows that there are point-like source there, or 0 when I am talking about things on the wall, because there's no point-like source there. Because the wall is actually blocking the light.

So it can be either 1 or 0. When is that equal to 1? When minus D over 2. If I define-- this is actually x equal to 0. The middle of the slit is actually x equal to 0. Then it is actually equal to 1 when minus D over 2 smaller than or equal to x, smaller or equal to D over 2. So that will give you a slit with width of capital D.

On the other hand, if the absolute value of x is greater than D over 2, then I get 0. So now you can see that is actually the meaning of f function. f function is actually giving you a map of the point-like source. And what I am going to do now is to really do the integration to sum over all the spherical electromagnetic waves coming from all those point-like source, and to calculate the total electric field.

So now I can go ahead and calculate C function, which is actually a function of kx and ky, related to the direction of propagation, or, say, the relative position of the observer and the overall point-like source. And this is actually equal to 1 over 4 pi squared, according to my formula. And now I'm going to do an integration from minus infinity to infinity.

But I found that there's a shortcut I can take. f x y is only nonzero between minus D over 2 and plus D over 2 in the x direction. Therefore, this integration becomes integration from minus D over 2 to positive D over 2 dx, exponential minus i kx times x. I'm taking part of the k vector dot r out of this formula. The relevant part related to x direction integration is exponential i kx times x.

And now I can actually do the integration in the y direction. So you can see that, in the y direction, this slit is infinity long, covering the from the left-hand side edge of the universe to the right-hand side edge of the universe. Really long. Super long.

Minus infinity to infinity in the y direction. The relevant part of the exponential minus k dot r is exponential minus i ky times y.

So, before I do this integration, I would like to remind you one thing which is actually we have learned from the past, from the help of math department. So we know delta function x minus a is actually equal to 1 over 2 pi, integration from minus infinity to infinity, exponential i p x minus a dp. So we know about this formula.

So that means I can easily evaluate this function. So this function, I'm actually doing the integration over y. Therefore, what I'm going to get is, I take 1 over 2 pi out of this. I take 2 pi out of this. Then, basically, I can actually arrive expression, which is actually delta function is a function of ky.

After you do this integration using this formula here. So p here is actually y in my integration I'm doing. And what I'm going to get is actually ky equal to 0, minus 0, and I simplify that to be delta function of ky. So basically, you are going to get the ky contribution is going to give you a delta function.

So how about the integration which is the other part of the integration? The other part of the integration is related to x direction, is here. So, basically, what is actually left over? I took already 1 over 2 pi from here.

Therefore, I have 1 over 2 pi, and I do the integration. It's just a exponential function. I'm not super worried. Basically, I get 1 over minus i kx. Exponential minus i kx x. And evaluated at D over 2, x equal to D over 2, and x equal to minus D over 2.

I hope this part is straightforward enough. Any questions so far? Everybody's following? All right. Very good.

So I will continue the red part. So I will just look at the red part and then continue on this board. I'm using the red pen, right. So, basically, what I am going to get is, basically you have 1 over 2 pi, 1 over minus i kx, exponential minus i kx D over 2, minus exponential i kx D over 2. So the red part of the left function become this.

And you can actually easily realize that this is actually proportional to a sine function, right? So basically I'm going to get 1 over 2 pi minus 2i sine kx D over 2, divided by i kx. This is actually coming from there. And this is actually coming from-- this minus 2i sine function is coming from the exponential function

I can cancel this 2, and basically I get 1 over pi kx sine kx D divided by 2. So if I put everything together, so basically what you are getting is delta function of ky, 1 over pi kx sine kx D over 2. Am I going too fast? Everybody is following?

So I hope this mathematics is straightforward enough. And don't forget what we are doing. So what we are doing is the following. So we have this two-dimensional Fourier transform. And the goal is to sum over all the waves coming from a shape defined by f function. And I'm going to evaluate the C function, and the C function is proportional to the total electric field.

C is a function of kx and ky. kx and ky give you the information about the direction, relative position of the source and the observer P. And from this exercise, what we actually learn from here is that the C function is a function of y, but essentially only nonzero when ky is equal to what?

AUDIENCE: 0.

PROFESSOR: 0, right? Does that surprise you? No, probably not. Why is that? Why should we expect that? Because in the y direction this slit is infinitely long.

So if you have contribution of many, many spherical wave, and this slit is infinity long, the sum of all those spherical wave is going to be still like a wavefront. You can do this in your head.

So that means the direction, if I choose a direction which is actually pointing to somewhere which is actually with a ky not equal to 0-- so that means I have a specific direction-- what I'm going to get is that the electric field, the total electric field, will be equal to 0.

And, of course, you can actually also talk about what will happen in the x direction. So that is actually the dependence of the C function to the kx. And we found interesting dependence. It's actually sine kx D over 2 divided by kx.

So what I'm going to do is to make our life slightly easier by defining something which is actually easier to understand. But before that, I would like to say that the electric field, as I mentioned before, is going to be proportional to the C function. And now I would like to drop the y direction, because it's just a delta function. Therefore, I can actually drop it in the discussion.

Then I will say that this electric field is going to be proportional to the sine kx D over 2, divided by kx. Since we have the electric field, the magnitude of the electric field, then I can actually calculate what will be the intensity. Intensity is actually what we care. It's going to be proportional to E square, and that is actually proportional to C square.

And what is actually that value? That is going to be proportional to sine square kx D divided by 2, divided by kx squared. Any questions so far?

So remember what is actually we are discussing. So we are discussing about a single slit, and we were wondering what will happen to observer point P when they actually do get, when these observer do get the interference pattern of all the point-like source between these two walls.

We can actually make it much more understandable by using angle, which is actually theta, which is the measure of AP, which is the direction of the-- which is a vector connecting the slit to the observer-- to the horizontal direction.

And I can define the displacement with respect to the center to be x. And I can actually also express AP by a vector which is r vector. Basically, after this definition, we can actually calculate or express sine theta.

Since the distance between the screen and the wall is very, very large, therefore the theta angle is very small. Therefore, I can safely assume that sine theta is actually x divided by r. And also, at the same time, this is actually equal to kx divided by k. Because the k vector is actually telling you the direction of propagation.

So, therefore, I can actually rewrite this. This will become kx. The magnitude of k vector is actually basically 2 pi over lambda. So, therefore, you can actually calculate that, and you will get kx times lambda divided by 2 pi.

Therefore, the goal is to rewrite kx in a form which we understand, which is theta. So now we have achieved that. What is actually kx? kx is actually equal to 2 pi sine theta divided by lambda.

And this means that my intensity, which I appended there, will be proportional to sine square pi D divided by lambda sine theta, divided by 2 pi sine theta divided by lambda, squared. So basically what I'm doing is to replace kx and then write it in terms of theta.

If I define beta to be equal to pi D sine theta over lambda, if I define this, basically you are getting sine square beta, this will be proportional to sine squared beta divided by beta squared. And this beta is actually proportional to theta and D.

Any questions so far? I'm just doing a replace, I'm just replacing the variables so that it's actually in terms of theta and in terms of some variable which actually simplify the expression dramatically.

So, that's very good. So we have actually evaluated the intensity, the resulting intensity which will show up on the screen. And then we found that essentially proportional to sine square pi D divided by lambda sine theta, divided by something squared.

And then I called this constant, sorry, I called this expression, I defined this expression to be beta. Then the functional form become much simpler. It's become sine square beta divided by beta square.

So what I am going to do now is to visualize this result. So what I'm trying to do now is to plot the intensity I as a function of sine theta, for example, using this expression.

So what I'm going to get is something which is actually going to be decreasing. Something is going to be decreasing as a function of beta. So that's the dashed line. This dashed line is actually proportional to 1 over beta squared.

And sine theta very small, you actually reach a maximum value of I0. When you move away from theta equal to 0, you actually will hit a minimum when the sine theta is equal to lambda over D. Because if sine theta is equal to lambda over D, then this expression become what? Become what value when sine theta is actually lambda over D?

Pi. Sine pi is 0, right? Therefore, you have a destructive interference. This point is really interesting. Why? Because that means all the point-like source, all of them between these two walls, are working together so nicely such that the total field is completely cancelled.

Isn't that remarkable? That's really, really crazy when this happens. Takes a lot of work, infinite number of source, to do that.

Then, if you actually increase further the sine theta, move away from the center of the screen, basically you see that this will increase again and reach a smaller maxima, and again reach 0 when this is actually equal to 2 lambda over D. And this pattern continues.

And, of course, because of the symmetry we observe in this expression, everything is actually proportional to sine squared something. Therefore, this distribution is actually symmetric. So you have minus lambda over D, minus 2 lambda over D, et cetera, et cetera. Any questions so far?

So what you can see here is something really interesting. Sine theta, if you multiply that by r, is telling you the position which you will see on the screen. So this is actually-- if you are interested in some place, point of interest P, and this actually just r times sine theta. And this is actually the slit. And I will move this thing closer here. And the size of this slit is called D.

So one thing which is actually very interesting in this result is that, if we look at the width of the central principal maxima. The width is actually the measure between the center and the first minima, where you have complete destructive interference.

What you actually see here is that this is actually something very interesting is happening. When you increase D, if you increase D, what is going to happen to the position of the first principal minima, of our first minima? It's going to what? Going to become smaller. Right?

So suppose I have a gap here and I'm shooting a gun like crazy, boo-boo-boo-boo boo-boo-boo-boo. And I produce huge amount of bullet, which I don't recommend to do that, for sure. What I'm going to do, what I'm going to get is a distribution like this, which are the bullets passing through this wall.

If I increase the size of the wall, the distribution I'm getting is becoming what? Wider. Right? But the result here is actually surprising. Why? When you increase the width, when you increase the width of the D, this function becomes smaller.

That means the central maxima will become narrower, as you can see from this demonstration. So the left-hand side is an experimental setup which you have a very, very narrow slit. And basically you get a very wide distribution in the intensity as a function of position on the screen.

Right-hand side is another situation where you have wider distribution. I'm sorry, wider slit, and you are going to get a narrower central maxima. Which is actually different from the other experiment which we were actually doing. So that's the first thing which we learn from here.

And, also, the distance, the distance between the maxima and the minima is proportional to wavelength. So that means I can measure wavelength by using the position of the minima. And we are going to do that to measure the wavelength of the laser beam.

And, finally, the last thing which we learn is that, in the central region, you have a maxima of I0, and this intensity is going to be going down, proportional to 1 over beta squared, where beta is actually defined here. It's proportional to D sine theta and inversely proportional to wavelength.

So now what I'm going to do is experiment which I would like to measure what would be the wavelength of my laser. So I have a laser here. Oh. OK, I don't want to hurt anybody.

So I have a laser here. And I have a slit, which you cannot see, unfortunately. And I can read off the width of the slit for you. The width of the slit is carefully designed to be really small, is 0.16 millimeter. This is my width. The D is actually equal to 0.16 millimeter.

And on the screen, you can see that there's a pattern formed here, which you probably cannot see very, very clearly, so I will try to lower the intensity of the other source. So you can see, then, see that there is an interference pattern or diffraction pattern which is actually showing here.

So what I really need in order to calculate the wavelength is the sine theta angle. Which I will really need the sine theta angle. Then I can actually calculate what will be the wavelength of this laser.

So that means I will need help from a volunteer. Who volunteer to help me to measure the distance between this slit and the large screen? Can somebody volunteer? Yes, please.

So we are going to measure the distance. Can you hold this? And can you actually put it? OK, try to pull this thing, and we will try our best to make it straight. Thank you very much. We don't want to destroy the experiment as well.

This is not working? Let me do this in the other way. So how about-- trial and error, right? How about this. You hold that thing, and I'm going to actually measure the distance from here. And I need to really make it really carefully, measure this very carefully. And I don't want to destroy anything, which is very possible.

So what I'm getting? I get 7.5 meter. So that's actually the distance between the screen, the screen and the source. Hold that for a second. I am going to measure the width of, the distance between two minima.

The distance between two minima is 7 centimeter. Thank you very much. Thank you for your help. So we have now everything we need to calculate the wavelength. I'm going to clean this up first.

We have what? We have the distance now. The distance between the source and the screen is 7.6 meter. So now I would like to calculate what will be the lambda. And also I know the distance-- the distance between these two minima is 7 centimeter.

So that means this will be 3.5 centimeter. So lambda divided by D is actually equal to sine theta. Which is actually small d, which is the distance between the minima. The small d is here. The small d is the distance between the minima and the center. Divided by r, which is the distance between the source and the screen.

Therefore, I can have lambda will be equal to capital D times small d divided by r. So what is actually the answer? So basically I have capital D, which is actually 0.16 millimeter. So that is actually shown there but you cannot see it.

So I will use a different board for this calculation. So, basically, we will actually get lambda is equal to capital D times small d divided by r. Capital D is 0.16 times 10 to the minus 3 meter.

And what is actually the small d? The small d is actually 3.5 centimeters. And, finally, I have 7.6 meter, which is actually the small r. Divided by 7.6 meter. Can somebody actually calculate this for me? Anybody have a smartphone?

This means that I haven't done this experiment myself, and we will see what is going to happen. I hope it will work. What is actually the value?

AUDIENCE: 7.368 times 10 to the negative 7.

PROFESSOR: 7.368 times 10 to the minus 7. This is actually equal to 7.37 times 10-- oh, wait. This is actually 737 nanometer. Actually, the wavelength of the red is actually between 620 and 750. And actually we are actually getting the correct value.

You see? So, actually, now you can actually tell your friends that, although the wavelength is so small, but I can't measure it with such a square feet experimental setup.

So that's a successful experiment. So that is actually telling you that it's a proof that this formula, which we actually do all the crazy work of this Fourier transform in two-dimensional integration, it should really work. And the result is actually not really far from what you can get from Wikipedia.

So, at this point, I would like to take a five-minute break to take some questions. And then we are going to come back in, at 31, and we are going to discuss another very interesting issue, resolution.

So welcome back. So there are a few questions about-- there were a few questions about the pattern here, which is interesting. So you can see that what we actually concluded from here is that the width of the central principal maxima is actually two times of the width of the secondary maxima.

So you can see that the width here between these two points is actually lambda over D. But the width between these two points, which actually give you the width of the central principal maxima, is actually 2 times of lambda over D.

And now this actually can be seen from the experiment there. Maybe not easy for the moment. But this is actually the width, and the smaller structure is actually having a width half of the central peak. So that is actually something which is interesting, and I would like to share that with everybody.

So now we actually come back to the original question we were actually discussing last time. So one interesting thing we observed in this two-slit interference experiment is that you not only see all those little structures, which is actually kind of periodic structure, and that they are coming from the two-slit interference.

And you also see this larger structure which is showing up there, which is actually going up and down, and also it produce minima at some specific point. Now we understand what is actually happening.

Suppose I have two-slit interference experiment, where I have the width of the slits to be capital D, to be very small. D is very, very small. And the distance between the slit is actually called small d. Which is kind of weird, but you have to accept that because it's on my note.

And you can see that, interestingly, if this is the situation, then you have this periodic pattern and you will see no decrease in amplitude as a function of distance with respect to the central point of the screen. So that's actually very nice.

However, if you consider a realistic situation, where the size, or say the width of the slit is not negligible, is sizable. And what is going to happen is that-- OK, let's forget about the second one for a moment. We already learned that the output intensity of a single slit is already varying as a function of angle. So I have this pattern.

Therefore, if you have these two realistic slit interacting with each other, have interference pattern, what you are going to expect is that you are going to have the two-slit interference pattern modulated by diffraction pattern. Because, originally, coming from a single slit, you already have a varying intensity as a function of sine theta, as we already discussed there.

So, if we put all those information together, we are going to get I. The intensity is going to be equal to I0, which is some maxima, sine beta divided by beta, square of that, sine N delta divided by 2, divided by sine delta divided by 2, squared.

So basically what I'm talking about is that, if you have N-slit experiment, each slit have the same width. And what you are going to get is-- this is actually the N-slit interference pattern. And that is actually modulated by diffraction pattern.

Where beta, just a reminder, in this summary is pi capital D divided by lambda sine theta. And the delta, which is the optical path length difference we defined before, is k times d sine theta, and that is actually equal to 2 pi times d sine theta divided by lambda.

So that is actually why, when we perform the experiment of a double-slit experiment in the last lecture, we get complicated interference pattern like this, and it has a very complicated structure. And now we actually understand why the structure is like this.

The small structure in this case is actually coming from interference, two-slit interference. And the additional structure, larger-scale structure, is actually coming from diffraction, is coming from the varying intensity of a single slit as a function of sine theta. Any questions so far? We are making a lot of progress.

So what I would like to move on is to discuss with you something really interesting. So we discussed and learned how to explain why we have actually colorful soap bubble. So I have something totally unrelated.

So we have a soap bubble also in the space, which is the Soap Bubble nebula. Which is really interesting, and you can actually Google it and see what is actually happening there. But, actually, that's actually not my point. Then my point is that you really need very good resolution telescope so that you can actually observe those really beautiful objects which are already there and cannot be made by human. Made by somebody else.

So this is actually what I'm getting into. So the resolution is really something important. So when you take a look at this picture, the resolution is not very good. So as you can see, now the peak position of two nearby peak is actually connecting to each other. Then what do we see from this picture?

You see maybe a lion? I don't know. Maybe, maybe not. But if you improve the resolution, what do you see? It's actually zebra. So this is actually the kind of thing which we would like to discuss with you. We are actually touching this important phenomenon, which is actually needed for observing an interesting phenomena which is actually happening really far away from the Earth.

What is actually the resolution? And we are going to talk about that as well. And I would like to show you another interesting example. So this is a comparison between not so serious picture and the picture from Hubble telescope.

So I was using some telescope with D equal to 40 centimeter. And that's actually the best thing which I can achieve, shooting the same planetary nebula M57. That object is actually 2,500 light year away. And you can see that I cannot get really a lot of detail from this image.

And now, if you compare that to D equal to 240 centimeter Hubble telescope, and also, at the same time, this thing is actually above the atmosphere. So that's actually very, very important. And you can see that you do get a much, much better resolution, and you can actually see all the fine detail, very, very fine detail of this image.

And we are in the position to understand the resolution and the limit which we can have due to diffraction, actually. So if I consider now a pinhole with diameter equal to D. So right now what we are actually doing is not a single slit any more, but a hole with radius D over 2.

And we can do the same, exactly the same calculation using this formula. But I'm not going to do that for the sake of time. So we can do exactly that same C function calculation.

And what we are going to get is I as a function of theta is equal to I0 J1 beta divided by beta, squared. Where J1 is the Bessel function of the first kind. Sounds really scary, but it's actually not. So what I really need is the zeros of the Bessel function, so that I can actually extract the interference pattern and the width of the central maximum.

So now, since we are having a pinhole, basically all of those things are, all those patterns are actually two-dimensional. And I was wondering what will be the needed beta value so that I can actually reach the minima. Why is that important? That is actually telling you the limit of the optical resolution.

If I have two peaks which are actually placed too close to each other, like what we actually see in the previous slide, then we can actually not separate very well these two light source. On the other hand, if the distance between these two peak is larger than the first minima, then I can actually be very safe. I can actually separate. I can say that, ha, this is really two peaks. Two stars, two light source. I can tell.

So that is actually why this is actually important. And where is actually the minima? And I can already solve that for you. And that is actually when x is equal to roughly, the numerical value is roughly 3.83.

So that's actually not important. Those numbers are not important. The important result is really the conclusion. So beta is equal to 3.83, and that is actually equal to pi D sine theta divided by lambda. So that is actually our original definition.

And I can solve what will be the sine theta, which is actually telling you the position of the minima. So sine theta will be equal to actually 1.22 lambda divided by D.

So what does that mean? That means the position where you have the first minima is actually happening when you have sine theta-- this is the theta-- when you have sine theta equal to 1.22 times lambda divided by D.

So that is actually very nice. Doing exactly the same exercise, and we now understand where my minima is. Then that is actually telling you something about the resolution.

So what I'm going to try to get into is that, now, let's design a phone together, a mobile phone together. So what is actually the width of the human pupil? The width is actually roughly 2 to 4 millimeter-- when narrow, when you see a lot of light all over the place-- or 3 to 8 millimeter. So that is actually the typical length when wide. So that is actually the width of the pinhole.

So, typically, the visible light, as we calculated, is something like 500 nanometer. And the width of the human pupil, we can actually take a number of 5 millimeter. And now we can actually try to give input to the phone design. So what will be the resolution if we take these two parameter together?

So, basically, the resolution of your eye, we can now calculate that. So what is that? That is actually 1.22 times 500 nanometer divided by-- OK, my function is D-- so divided by D is equal to 5 millimeter. 5 millimeter. Basically, what you are going to get is 1.22 times 10 to the minus 4. This is actually the resolution. Sine theta, roughly equal to theta, is actually equal to 1.22 times 10 to the minus 4.

I have a iPhone 6 or 7, whatever you have. Basically is 401 ppi. 401 ppi is actually what is that? Pixel per inch. So what is actually the delta x?

So if I have a phone, OK, it has a camera there. That is my phone. And this is my eye. Looks like an eye. The distance is 20 centimeter. I do this, which is unusual.

We have 400 ppi. So what is actually the delta x, the delta x between the pixels? The delta x is equal to 2.54 centimeter divided by 401, and that will give you something like 6.3 times 10 to the minus 3 centimeter.

If I am trying to be healthy and I do this, then what is actually the delta theta? The delta theta is delta x divided by 20 centimeter, and that is 3 times 10 to be minus 4.

If you compare this value to the limit I calculated here, you can see that, what is the conclusion? Can I resolve the pixels on the phone? The answer is yes. So that means this phone is not good enough. They have to do more work.

And now I'm going to design a jPhone. Maybe at some point I got really crazy and I decided to open a company, which is Yen-Jie's phone company. And, of course, I will say this is jPhone because it's Yen-Jie.

And I'm going to put 40,000 ppi in this phone. Will you buy it?

AUDIENCE: No.

AUDIENCE: Sure.

AUDIENCE: How much?

PROFESSOR: $1. You'll buy it? We'll see. Maybe you will buy it because you are my student. But it's not worth it. Why is that? Because you cannot resolve this kind of fine or small distance between pixels. So it's actually useless.

So what is actually the limit? You can also probably give that to your friends. 2,000 pixel per inch is roughly the limit. Beyond that, maybe the next generation of our students will be using this like this. Then it works, and it is worth it. You can actually read this distance. It doesn't work for old people like me, but for young people it works.

So very good. So that's another thing which you have learned. So, finally, as I promised you, we are going to go back to this business of designing the Enterprise for Star Trek. So what does Enterprise do to their friends? They shoot laser beam. And they try to attack the other ships.

And what I'm going to do now is to calculate for you what is going to happen. OK, now I have this laser beam here. And, in principle, before you take 8.03, you are going to say, aha, I can shoot the moon. And this light is going to be really narrow and it's going to hit the moon, a very small area on the moon.

Do you believe that now? I hope the answer is not. How crazy is this idea? What is the size of the spot? Can you guess?

Is that 1 millimeter? 10 meter? Or 200 kilometer? How many of you think by now is 1 millimeter? Nobody? Fortunately.

How about 10 meter? One, two, three. OK. Three of you. How about 200 kilometer? You believe that? Really? The answer is really 200 kilometer. It's the size of Missouri state.

So now you can see that this is not practical at all, and you have to really do what? Increase or decrease the radius?

AUDIENCE: Increase.

PROFESSOR: Increase. By the end of this lecture everybody get this idea. Thank you very much for the attention. And I hope you enjoyed this lecture.