

Massachusetts Institute of Technology  
 Physics 8.03SC Fall 2016  
 Homework 9

## Problems

### Problem 9.1 (20 pts)

The ionosphere can be viewed as a dielectric medium of refractive index

$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

Where  $\omega$  is the frequency and  $\omega_p$  is the plasma frequency assumed to be a constant.

- Make a plot of the magnitude of the propagation vector  $\vec{k}$  as a function of frequency  $\omega$  for an electromagnetic wave propagating through the ionosphere.
- Calculate the phase velocity and group velocity of a radio wave propagating at frequency  $\omega = \sqrt{2} \cdot \omega_p$ .
- What happens to an electromagnetic disturbance within the ionosphere when  $\omega < \omega_p$ ?

### Problem 9.2 (20 pts)

Consider two polarizers and a quarter wave plate as shown in the Figure 1.

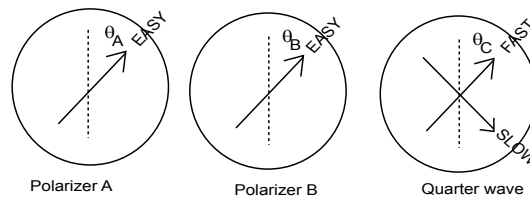


Figure 1: Two polarizers and a quarter wave plate

The two polarizers  $A$  and  $B$  are first inserted in a line of a light beam as shown in the Figure 2. The incoming light beam is moving along  $z$  axis and it is linearly polarized along  $y$  direction. The initial light intensity is  $I_0$ . In this problem you are asked to determine light intensity relative to  $I_0$  along the line of the beam. The polarizers are perfect in that they transmit 100% the light polarized along their “easy” axis and stop all light perpendicular to that axis. The angles of the easy axes of the two polarizers ( $\theta_A$ ,

$\theta_B$ ) are measured with respect to the  $y$  direction. Note: light intensity is proportional to time average of  $\vec{E}^2$ :  $I \propto \langle \vec{E}^2 \rangle_t$ .

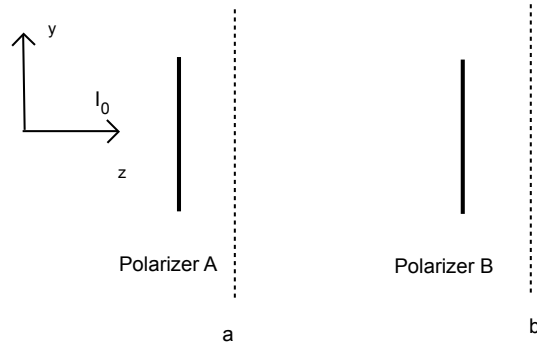


Figure 2: Two polarizers

- Find light intensity  $I_a$  of the beam after passing through polarizer  $A$  as a function of  $I_0$  and  $\theta_A$ .
- Find light intensity  $I_b$  of the beam after passing through both polarizers as a function of  $I_0$ ,  $\theta_A$  and  $\theta_B$ .

Polarimeter A is now replaced by a quarter wave plate  $C$  (see Figure 3). The quarter wave plate affects only the phase of the transmitted light. It has two perpendicular axes “slow” and “fast”. The phase of the light polarized along “slow” axis is delayed by  $\pi/2$  with respect to the phase of the light polarized along “fast” axis. The quarter wave plate is positioned such that the angle of the “fast” axis to the  $y$  axis is  $\theta_C$ .

- Find light intensity  $I_c$  of the beam after passing through quarter plate  $C$  as a function of given parameters.
- The intensity of light passing through the polarizer B will be affected in general by the presence of the quarter plate. Assume that the easy axis of polarizer B is oriented along  $y$  axis ( $\theta_B = 0$ ). Find light intensity  $I_b$  of the beam after passing through polarizer  $B$  as a function of  $I_0$  and  $\theta_C$ .

### Problem 9.3 (20 pts)

A plane, monochromatic electromagnetic wave travels in the  $+z$  direction within a dielectric medium. The wave is linearly polarized with the  $\vec{E}$  field at  $30^\circ$  to the  $x$ -axis. The angular frequency of the wave is  $\omega$  and the dielectric constant of the medium is  $\kappa_e$ .

- Write expressions for  $E_x(z, t)$ ,  $E_y(z, t)$ ,  $B_x(z, t)$ ,  $B_y(z, t)$ , in terms of  $\omega$ ,  $c$ ,  $\kappa_e$ ,  $E_0$  where  $E_0$  is the amplitude of the electric field.
- What is the rate of energy flow per unit area, per unit time which is transported across a surface perpendicular to the direction of propagation?
- What is the wave equation for electromagnetic waves traveling within the medium?

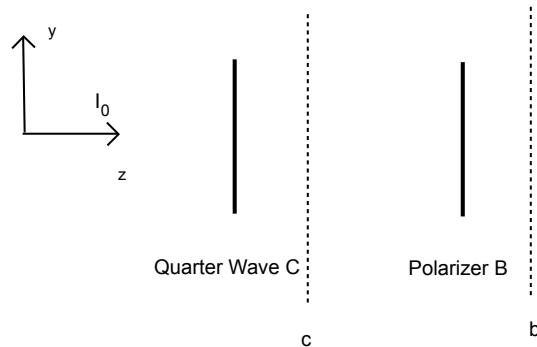


Figure 3: Quarter wave plate and a polarizer

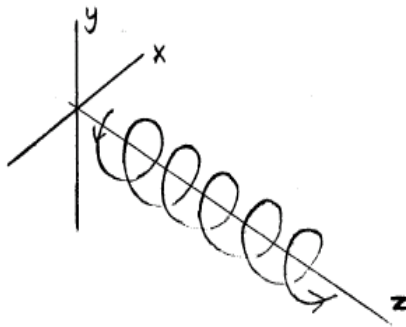


Figure 4: Spiraling electron

### Problem 9.4 (20 pts)

An electron executes spiral motion in a magnetic field as shown in Figure 4. Its speed is non-relativistic. The position of the electron is given by:

$$\vec{r}(t) = \hat{x}\alpha \cos(\beta t) + \hat{y}\alpha \sin(\beta t) + \hat{z}\gamma t$$

- Give the frequency  $\nu$ , the wavevector  $\vec{k}$  and the polarization state of radiation observed by a radio astronomer located far away in the following directions  $(x, y, z)$  ( $R \gg \alpha$ )
  - $(0, 0, R)$
  - $(R/\sqrt{2}, 0, R/\sqrt{2})$
  - $(R, 0, 0)$
- Give the complete expression for the electric and magnetic field vectors of the waves observed at point B on the  $z$  axis with coordinates  $(x, y, z) = (0, 0, R)$  where  $R \gg \alpha$ .
- Give an expression for the total energy radiated per second by the electron in terms of given constants. (Hint: you can use Larmor formula: Power  $P = \frac{q^2 |a|^2}{6\pi\epsilon_0 c^3}$ )

- d. On a sphere of radius  $R$  about the source, what is the ratio between the largest and smallest value of the time averaged Poynting vector?

**Problem 9.5 (20 pts)**

A light ray containing electric field vector components parallel ( $\vec{E}_{\parallel}$ ) and perpendicular ( $\vec{E}_{\perp}$ ) to the plane of incidence of a glass-air interface is shown in Figure 5.

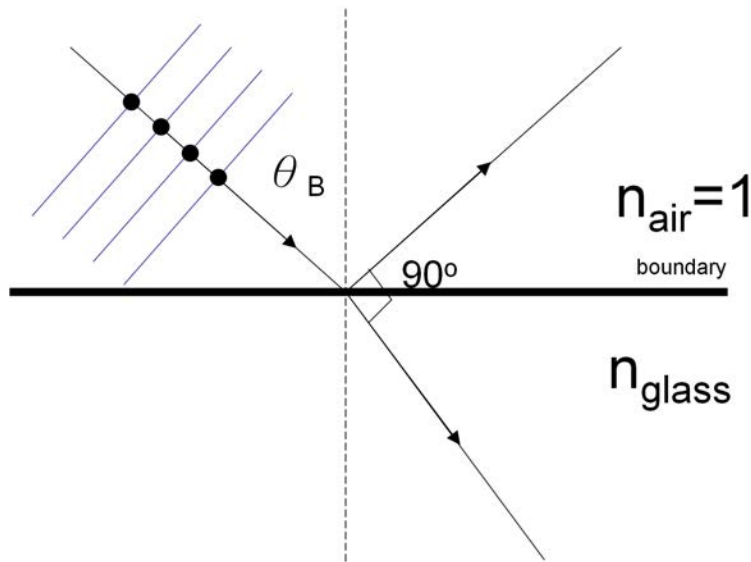


Figure 5: Glass-air interface

- If the ray is incident on the glass at Brewster's angle, show by means of sketches (like what is done for the incident ray in the Figure 5.) the polarization components of both the reflected and refracted rays.
- Give a defining equation for the Brewster angle  $\theta_B$  (Hint: use Snell's law)
- Explain on the basis of the acceleration of charges, in the glass, the direction of polarization of the reflected light. Why is the reflected light polarized parallel to the interface?

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