YEN-JIE LEE: All right, so let's get started. So today, very happy to have you here. We are going to talk about continuing our discussion on the two-dimensional and three-dimensional, waves. So as I mentioned before, there are two interesting situation which we can actually increase the dimensional.

So for example, I can have all the objects oscillating in just one direction, but I changed the way that I placed all those objects in the space. For example, I can have particles which are arranged in two or three-dimensional arrays.

And we were talking about how to understand this kind of system. And all those objects are oscillating in just one direction-- for example, up and down in one direction. That's actually one measure we can increase the dimension.

There's another interesting example which we will talk about today is to change the direction of the oscillation, even rotate the natural electromagnetic waves, for example. And how can we achieve that and how do we understand this phenomena, that's actually going to be covered by the lecture today.

So before we move forward, we will have a short review on what we have learned from the last lecture. And this is actually what we discussed. If I have two materials, the left-hand side and the right-hand side are two different kinds of materials, or very thin membranes.

And if I have an incident wave coming into the boundary of these two materials, which is just x equal to 0, as we discussed last time, basically the boundary condition require that in order to make sure that the membrane doesn't break, that means the k vector, the projection in the y direction of the k vectors has to be the same.

That means the projection of the wave in the y direction, which is along this line, the wavelength of all the incidents refracted and transmitted wave, the wavelengths should be all the same. Otherwise, I can change y and make the membrane break, or break the boundary condition. So that's actually the first thing which we learned from the math we were doing last time.

And also, the k value are not arbitrary, right? So we were already assuming that all three waves, all the three plane waves are oscillating at the same frequency. If they are oscillating at different frequencies, it doesn't work because I can change time, then make the membranes break. So therefore, all the three plane waves will be oscillating at the same frequency-- omega.

And according to this formula, so we have defined and which is actually the refraction index, which is equal to some constant c divided by the speed of the propagation, the phase velocity of the left-hand side material.

And we also can define n prime, which is actually equal to c, the same constant, divided by v prime, which is the speed of propagation of the right-hand side material. And according to the dispersion relation, we can calculate what will be the length of the k if omega is given. The omega for the left-hand side and right-hand, refracted, transmitted, and incident waves should be the same.

Therefore, I can immediately write down what would be the length of the k vector. And that means the length of the k vector would be equal to n times omega over c. So c over n is actually just v. So basically it's just the non-dispersive median dispersion relation. And also you can go ahead and write down what would be the length of the k prime. And that is actually determined by n prime omega over c.

So based on these two interesting formations, what we can conclude is that that means k prime and kr cannot be arbitrary. They have to be aligned to form a specific pattern. The pattern is that the projection in the right direction should be the same. And also at the same time, the length of the k vector is determined by the refraction index.

So once we fix all that and put them all together, we can conclude that based on the projection in the y direction, we will conclude that theta will be equal to theta r, where theta is actually that incident angle and theta r is the refraction angle, which is describing the direction of the refractive wave.

And also the second very interesting information we learned is that based on the boundary condition, we can conclude n sine theta will be equal to n prime sine theta prime. So that's actually Snell's law, which some of you actually learned from the high school days or from the earlier lectures in physics. And these two interesting results form the basis of geometrical optics or laws. That's two of the most important laws we learn from geometrical optics.

So that actually gives you some examples and gives you some more feeling about what we are talking about. So what we are talking about is that if I have an incident wave coming into this boundary, and that incident angle is theta 1, what I would expect based on what we have just derived is that theta r, the refracted wave direction, the refractive wave angle, the refraction angle would be equal to the incident angle, according to what we have just derived.

I was just rotating this by 90 degrees. And also we can calculate what will be theta 2 according to Snell's law-- n1 sine theta 1 equal to n2 sine theta 2. And if I continue and propagate the incident wave, here I assume that n2 is larger. If you have a larger refraction index, that means the speed of propagation is smaller. Larger n value give you a smaller speed of propagation. Therefore, you can see that the wavefront, which is actually the position of the peak, actually got delayed compared to the original projection.

And you can see that the red line is actually really what you would see in the second median. And then we can also continue to propagate and you will see that, interesting, that means the plane wave would change direction because of the matching boundary condition, the membranes doesn't break. You can see that the peak position match from the median number 1 and median number 2. The peak position, which is the position of the line, match.

And also, due to the slower speed of propagation, the plane wave actually change direction. So that's actually how we can understand the mathematical result which we derived last time by this interesting example. So that's considered a situation which is maybe a little bit interesting to you.

So what will happen if I now shoot light from inside some material which delay the light slightly-- for example, n1 is equal to 1.5. I shoot something through the material, and then the second material I have has higher speed of light, which is actually having the speed of light equal to light in a vacuum case.

So in this example, I have n1 equal to n2. Since n1 sine theta 1 is equal to n2 sine theta 2, so now I can calculate what will be the resulting theta 2 according to this equation. So the resulting theta 2 will be sine theta 2 will be equal to n1 over n2 sine theta 1.

In this case, n1 is greater than n2, therefore, this term is greater than 1. In this case, actually, this ratio is equal to 1.5. So this is actually pretty interesting. That means this factor is actually greater than 1. I have sine theta 1, which is multiplied in this vector.

So that means if I increase theta 1, if I increase theta 1, at some point I will not be able to get theta 2 because theta 2 will be arcsine 1.5 sine 1. And at some point, 1.5 sine theta 1 will be greater than 1, then I don't have a theta 2, which can satisfy this equation.

So what will happen? Maybe the whole system explodes, I don't know. So we are going to do a simple experiment to see what is going to happen. So before we do this kind of dangerous experiment, we are going to turn this light off and be prepared. Hope everybody will survive.

So here I have a laser, which is actually shooting a laser beam through this tank of water. So this is water, with n value roughly 1.33. And outside of water is air, so the speed of the light is roughly equal to c. So n value is actually roughly 1.

So that's actually exactly the situation we are looking for. And let's turn on this experiment, careful. Wow, look at this. You can see that here there was no light coming out. Why is that?

Because of the mathematics-- the mathematics say that, OK, sorry, guys, theta 2 doesn't work. Therefore, there will be no light really coming out. You can see my hand, you can see the light here.

When I put my hand here, it's not burning my hand. But that's OK. All right, and you can see that nothing really comes out. And all the energy are bounced back into the water.

The same thing also happened here. You can see that the light is actually bouncing back and forth and moving into the left-hand side direction until it passed through here, maybe into there, because it's actually still bouncing around inside this tank.

So the good news is that no explosion, like what we have here. The interesting thing we find is that all the energy will be bounced back if you have large enough incident angle in this situation.

As you can see from here that all the light's energy are bounced back into water. So that's actually very, very interesting. And that means we can probably make use of this property to send light through some large n material. So that's actually how optical fiber works.

So basically you can actually shoot a light into the optical fiber. And the light is going to be bouncing back and forth between the boundaries inside the fiber. And then you can actually send those information through light by this kind of mechanism. So that's how optical fiber actually works.

And we have a setup, which I have a light here, shooting light into the optical fiber. And I have paper here which tried to block part of the light. So from here, you can see the text which I put on the paper, because some of the light is blocked by this paper, the text on the paper. And this light goes through this optical fiber and continues and propagates and got captured by the camera.

So you can see, can you read out loud what is actually written there? Can you see it? The--

AUDIENCE: Last question on the

YEN-JIE LEE: The exam is-- oh, no, no, no, no, transmission interrupted. Oh my god. All right, so I'm sorry it didn't work, but it worked in the beginning, right?

So you can see that this is a wonderful way to send optical signal. And we actually do that. We send signal from US to Asia through all those crazy lines under the sea. So that's really cool.

And finally before we change the topic, I'm sure that you will enjoy doing the practice in your PSET number 8. We are going to learn that this is actually highly related to a beautiful phenomena we see in daily life-- the rainbow is actually really related to Snell's law, and that you are going to solve this problem in the PSET.

All right, so any questions so far related to Snell's law and refraction index, et cetera? All right, so if not, we'll go ahead and talk about the second example. The second example is that instead introducing more objects in this array, we could change the direction of the oscillation as a function of time and see what is going to happen.

For example, the direction of the electric field, I can make it rotate or change as a function of time. And that's actually called circularly polarized wave. And that means we are going to talk about polarization today.

Before we start the real lecture, what we are going to do is that I will quote words from Feynman. "It doesn't matter how beautiful your theory is, it doesn't matter how smart you are, but if it doesn't agree with experiment, is wrong." So that's actually a very important lesson.

And I have been telling you that electromagnetic wave is predicted to be oscillating in the transverse direction, as you can see from that little figure. So if the directional of propagation is to the right-hand side, so that means electric field and the magnetic field can only be oscillating in a plane which is perpendicular to the direction of propagation. That's actually what we have learned from Maxwell's equation.

But all those things are theory, right? Do you believe those theory? I'm not sure. Maybe light is actually oscillating in the longitudinal direction, right? If you are a physicist, you should ask this question. Maybe you'll find something which is inconsistent with the theory, then you'll feel really happy.

So the question we are trying to answer is how do we know the electromagnetic waves are transverse waves. And we are going to work through a few examples to convince ourselves, maybe that's the case, maybe we have some hint. Of course, we cannot prove 100% but very likely this is probably the case.

So let's go ahead and start the discussion today. So instead of adding one more dimension by arranging materials, we can actually discuss how the pointing direction of the electric field depends as a function of time. So in order to do that, I need to remind you how to write down electric field for the electromagnetic wave.

So in a previous lecture, if I organize myself so that the electromagnetic field is going in the z direction-- the electromagnetic field is going in the z direction. All right, if I choose that, then basically electric field as a function of z and time will be equal to the real part of some vector psi 0 times exponential i kz minus omega t.

If I assume that this electric field, the electromagnetic field is propagating to where-- the z direction, right? So by now, you should get used to this already. This is going to the positive z direction. Where I intentionally write psi 0 there is actually a vector which contains two components-- psi 1 in the x direction and psi 2 in the y direction.

And you can see from here that you can immediately recognize that this can be returned as a superposition of two waves. So one is actually electromagnetic wave with electric field in the x direction. And the other wave, the other electromagnetic wave, is actually oscillating with electric field in the y direction and with amplitude psi 2.

So that is one you can get immediately because of the superposition principle. So it's a superposition of two electromagnetic waves. And of course, in this case, I take the real part of this vector. So in general, psi 1 can be a1 exponential i phi 1. And in general psi 2 can be a2 exponential i phi 2.

So in this notation basically, we write everything in terms of vector. And sometimes we write those things in terms of matrix, and that sometimes serve a better purpose for calculation. So we can also rewrite this thing in matrix form.

So I have a matrix E which have two components, E x and E y. And this is actually equal to a real part of some Z matrix, which also contain two components, times a scalar, which is exponent i kz minus omega t.

So as can see, I am now just setting up the language we want to speak so that we communicate. So in this case, the z matrix is written as psi 1 and psi 2, which are the two components. One is in the x direction, the other one is in the y direction.

And we are going to use this language and see what we learn from it. So you can see that we have been discussing electromagnetic wave, which is propagating toward the positive z direction. And I have two component which I can have-- x direction electric field and y direction electric field.

So let me try to go through some example and see what we can actually learn from this. So the first example I would like to talk about is that if I have two waves, E1. The first wave is actually E0 cosine kz minus omega t in the x direction.

And then the second wave is E2 equal to E0 cosine kz minus omega t, in the y direction. Be careful-- the direction of the electric field is different for the first and the second plane wave. And if you actually notice from this expression, these two waves are in phase. So in-phase means that they reach maxima at the same time.

So in this case, if I add two waves with no phase difference, what is going to happen? So I am going to have a E vector, which is E1 plus E2. And if I plug the locus of this kind of electric field in the two-dimensional space, x and y, so now I am plotting, so I am fixing my z position for example at 0. And I would like to see how the locus of the E vary as a function of time.

In this case, since E1 and E2 have the same amplitude, which is E0, and also they have no phase difference, that means they reach maxima simultaneously. So that means originally, for example, in the beginning the electric field is 0. And this electric field projection to the xy plane will increase until some maximum. At the maxima, this will be E0, the x and y position will be E0. Then it goes back to minus E0, minus E0, in the x and y direction position. And it goes back and forth.

So in this case, this is actually still not very different from what we had discussed before because the electric field is still oscillating up and down. But the difference is that it's not oscillating at the x-axis or y-axis, but in the axis which is actually 45 degree with respect to the x or y-axis.

And now we can do a exercise to write it down in the matrix notation. Now I have this E matrix will be equal to the real part of E0, E0. And both E0's are real, exponential i kz minus omega t. And basically I can conclude that z will be equal to E0, E0, and this will be E0, 1, 1.

In this kind of situation, the electric field is varying as a function of time. The position to xy plane is a line. When I happens, we call it linearly polarized.

So this is actually just a name. But indeed the locus of the electric field on the xy plane is a linear line, so that we call it linearly polarized.

And of course you can say, OK, this is just one example. I can have many, many examples which you also create a line when you plot the locus. So for example, I can have z equal to E0, 1, 0.

Can somebody tell me what does that notation mean?

AUDIENCE: That just means one wave electromagnetic wave in one direction.

YEN-JIE LEE: That's right. So that means you only have electric field in the x direction. So this is the x direction, this is y direction. And this is also linearly polarized.

Of course you can have very similarly E0, 0, 1. And in this case, you only have electric field in the y direction. Just want to tell you my language I'm introducing here so that we communicate.

And of course, you can have z equal to E0 cosine theta sine theta. What does that mean? This means that when I plot the locus on the xy plane, I'm going to have a straight line still, but now the angle between this line and the x-axis is going to be theta.

So all those examples, you can see that in the first case the oscillation is in the x direction. And in the second case the oscillation is in the y direction. And then in the third case it can be in an arbitrary line-- theta angle away from the x-axis. So all those things are linearly polarized.

So so far, there is nothing to surprise you. So that's what we have been talking about. And you can say that, Professor Lee, you were just not doing a very good job. You didn't rotate the axis right so that all the electric field is in the x direction. So in principle for the discussion of the linearly polarized wave, you can do a good job by rotating your axis so that you only have one component in the x direction. So I say, OK, yes, I agree, but this is useful discussion.

All right, so now maybe you're bored. They have no phase difference, right? So how about we introduce some phase difference and see what will happen. So now if I consider the second situation-- I have E1 is equal to E0 cosine kz minus omega t in the x direction.

Now I consider E2, this would be E0 sine kz minus omega t in the y direction. I hope you can see it. And then you can see that now they reach the x component and y component electric field. They reach maxima at different times because of the phase difference.

How big is the phase difference? Can somebody actually remind me?

AUDIENCE: Pi over 2.

YEN-JIE LEE: Pi over 2, very good. All right, so this I can write it down as E0 cosine kz minus omega t minus pi over 2, as you already figured out. And now I can also again write it in the language I like, the matrix format. Before I achieve that, I can write it as a real part of E0 in the x direction plus E0 exponential minus i pi over 2, because this minus pi over 2 sign, the factor here for the y direction.

And all those things are multiplied by exponential i kz minus omega t. Both of them are actually oscillating at the same omega angular frequency. And you can see that now I can collect this phase difference back into a complex factor here, exponential minus i pi over 2. And exponential minus i pi over 2 is actually just minus i. So basically you can figure out that it's just minus i.

So then I can write down my expression for the E matrix. This one look like real part E0, 1, minus i, exponential i kz minus omega t. And in this case, z vector will be 1 minus i in the language which I introduced today. Everybody is following?

Am I going too fast? Very good. Thank you for the feedback. So now I'm going to do the same thing to plot the locus of the electric field as a function of time on the xy plane.

So now you can see that it's pretty interesting. Since I have a cosine and sine, I assume that t is equal to 0, z is equal to 0, then I will only have electric field in the x direction. So this is actually when t is equal to 0.

All right, and if I increase time, I fix the z to be equal to 0. If I increase the time, this locus is going to do an interesting thing. It's going to be rotating until it finish one period.

Why is that happening? Because what I'm plotting is the locus of the electric field. And you can see that if I set z equal to 0, when z is equal to 0, this will give you E0 cosine omega t in the x direction and E0 sine omega t in the y direction.

So E0 cosine omega t, E0 sine omega t-- wow, this reminds you about the previous discussion. If you have a cosine and a sine, they work together in different directions, that's going to get you a circle. So that's why in this locus you see a circle.

And it's rotating in a counter-clockwise direction. And the speed of the rotation is related to omega t. And now if I increase the time, then this point is going to be going in a counter-clockwise direction.

And you may be super surprised because what I have been doing is to add two linearly polarized waves together. The only thing which I always say, the magic I have been doing, is to introduce a phase difference.

And you can see that instead of going up and down, up and down as a function of time, now it's actually doing rotation. So what is going to happen is that as a function of the time the direction of the electric field is going to rotate as a function of time. So we call this situation circularly polarized. By the way, because of the initial two components I put in, both of them have amplitude E0. Therefore, it's circularly polarized.

I can also try to do something different. For example, I can change the x direction. So the third situation is that I can change the first x direction amplitude from E0 to E0 over 2. Then what I'm going to get is something like this. The only thing which I change with respect to 2 is that I changed the amplitude of the electric field in the x direction by a factor of 2.

Can somebody actually tell me what is going to happen in this situation? What will happen to the locus?

AUDIENCE: Gets squished.

YEN-JIE LEE: Yeah, it gets squished in, so in the x direction. Yeah, that's right, very good. So what you are going to get is that instead of a circle you get something like this. So this is the x direction and this is the y direction.

And this will give you a maxima x equal to E0 over 2. And of course the original amplitude in the y direction didn't change, and that gives you a maximum value of E0. And this becomes an elliptically polarized wave. So this kind of situation is called elliptically polarized.

How do we actually visualize this situation? So that means, as in the beginning, when the electric field is aligned with the x-axis, it's pointing in the x-axis. It has a shorter length.

And when it rotate, rotate as a function of time to the y-axis, the amplitude becomes bigger. And it gets smaller again, and increase again, it becomes bigger. So you can see the amplitude is actually changing as a function of time. When you have this kind of situation, they have different amplitudes, although they have a fixed phase difference, so pi over 2. Any questions so far?

OK, so if you noticed that there's actually another way to produce elliptically polarized wave. What we could do is that instead of changing the phase by a factor of pi over 2, we can change the phase difference. The phase difference can be delta phi, which is the phase difference, can be a different value, arbitrary value, not equal to pi over 2 or 3 pi over 2.

If the delta phi, which is the phase difference between x and the y direction, EM waves-- electric field. If the phase difference is not pi over 2 or not 2 pi over 2, then you can also create an elliptically polarized wave.

So that starts from the original figure, which we actually discussed-- situation number 1. Situation number 1, I have delta phi, the phase difference equal to 0. When that happened, you have a linearly polarized wave. Basically what you are going to get is a line in this two-dimensional space.

If I increase, so now I slightly delayed. If I slightly delay the electric field in the y direction. So if I slightly delay electric field it in the y direction so that delta phi now is greater than 1, what is going to happen is that it's going to look like this.

This means that they will not reach maxima really simultaneously. There will be because of the phase difference. And you'll see that originally when there were no phase difference, you would be oscillating back and forth in this blue line. When you increase the delta phi slightly, then you are going to get also a elliptical shape, but now is tilted with some degree, which is 45 degree in this case.

So therefore you can see that now I can also create this is also elliptically polarized wave, I can also create elliptically polarized wave by adding two components, which they have some slight phase difference. So that is actually how I can create a something which is called circularly polarized or elliptically polarized.

Originally before you come into this class, it may look really completely bizarre that, oh, I can have electric field going up and down as a function of time. But how could I rotate this thing, right? Looks really strange. How can I see this from the Maxwell's equation?

But now you get the idea. Basically, that's because I can now overlap two components. Both components individually are linearly polarized.

But if I introduce a phase difference, then the superposition of these two components become something which is actually rotating as a function of time. And that's pretty interesting. So let's visualize what we have learned so far.

The first one is linearly polarized. Actually, it doesn't surprise you-- that's the example which we have been using in the previous lectures. And this is the situation of circularly polarized wave.

So let's focus on the figure at z equal to 0. So you can see that in this case the direction of the electric field is actually rotating as a function of time. And of course as we discussed, I can actually add two electric fields with different amplitudes or introduce slightly different phase, then you will see that not only that the direction is changing, but also the amplitude is changing. And in this case we call it elliptically polarized wave. Any questions so far? Yes?

AUDIENCE: So the magnitude always is constant with time now?

YEN-JIE LEE: Yes, so the magnitude, or say the intensity, is proportional to E squared, right? So it's actually a constant.

So now let me add more excitement here. So now suppose I have a perfect conductor, all right. So what I am going to do is that I'm going to introduce some more excitement by shooting this linearly polarized wave through some material. If I have a perfect conductor where all those strips, I have only strips of perfect conductor instead of a plate-- we were talking about plate before.

And the lesson we learned is that the plate is going to refract the electromagnetic wave because that electrons on the plate is so busy, it's trying to make sure that the electric field in the surface of the plate is equal to 0 because that all those electrons are really moving around. So that's making sure the electric field is canceled. And therefore, it's going to refract the electromagnetic wave.

So how about I restrict the direction of the movement of the electron so that it can only move in the horizontal direction? What is going to happen is that in this case, the electric field is actually oscillating up and down. And the electrons see this field and they will say, no, no, no, this is not why we work. This is not we are going to vote for.

And I'm going to rearrange ourselves to compensate that, all right. And they were asking, can I move. Oh yes, I can move in the horizontal direction. So they jump up and down, then they can actually cancel this electric field.

Therefore, what is going to happen when we have this perfect conductor is that the electromagnetic field is going to be bounced back like what we had before when we talked about a metal plate, all right. So that's actually pretty nice.

Now suppose I have another perfect conductor. If I have another perfect conductor, this time the perfect conductor is arranged such that all those charges can only move inside or outside of the board instead of going up and down.

Now I have the incident wave, which is actually polarized in the horizontal direction. So you can see that this time the electrons are really nervous about this. And oh no, I have to do something, but they cannot move up and down. Therefore, what is going to happen is that there will be no cancellation of electric field at this boundary. And this polarized wave can pass through this so-called polarizer or perfect conductor without getting stopped or get refracted.

And as you can see that in these two examples, I have so called the easy axis. So you can see that the easy axis, as the name should tell you, so easy axis is the axis which is easy. All right, so what does that mean?

That means if I have electric field aligned with the easy axis, it will pass through. And the easy axis is perpendicular to the direction of all those streets I have in the perfect conductor.

Let's look at the first example. It tells you that the electric field is not aligned with the easy axis. That means it's not easy, so it got refracted-- oh, life is hard.

And the second electric field is smarter. It got aligned with the easy axis. So life is easy, it passed through. That's how I remember this so-called easy axis. Any questions?

So we have been talking about all those crazy names of polarized light. And I hope that you also have learned about unpolarized light before. Most of the lights are unpolarized.

For example, light emitted from those light bulbs are not polarized because every emission of a photon can be aligned in different direction. Therefore, as I was talking about, there are an infinite number of photons hitting my face and bouncing back to your eye. All those things are not aligned to the same direction as what we have been discussing before.

In the case of linearly polarized wave, all the electric fields are pointing to the same direction. And apparently those lights hitting my face is not aligned and it's actually pointing to random directions.

So that brings me to the fifth situation, which I would like to talk about. The fifth situation is that I could have unpolarized light. So what do I mean by unpolarized light? That means electromagnetic waves which were produced independently by a large number of uncorrelated source.

You may ask, OK, wait, wait, wait-- unpolarized means that we have a lot of different light source pointing to different angles. Shouldn't it just look like this? So you have pointing this direction, all the directions, all the possible directions, pointing to all different possible directions, like this.

Shouldn't that diagram actually tell you something about unpolarized light? Life is hard. If everything is aligned and also arriving at the same time like this, what is going to happen? They cancel, right? Because all those things are vectors.

If they reach maxima at the same time, this means that they are emitting always at the same time, but in random direction. And emitting at the same wavelength, then they are going to cancel each other. So apparently that's not what I mean by unpolarized light.

What I mean is that they can be pointing to different directions, but that emission time and the wavelength of all those emitted electromagnetic waves can be slightly different, otherwise they will just cancel each other completely. Any questions about what I just said?

So very good. So now what I am going to do is to show you some examples related to polarizer. Before that, I will take a five minute break before we come back to the demonstration. So we will come back at 36.

OK, so welcome back. So we would continue the discussion of the polarization and the polarized wave and the polarizers. So before that, I already introduced the polarizer by the perfect conductor arranged in the x direction or y direction, many of them are strips, aligned in the x direction and the y direction.

And usually we use a simple diagram like this, a circle and an arrow, to indicate the easy axis to tell you about the property of the polarizer. So in this case, if I have a circle and I have an arrow, which is pointing toward the x-axis, in this case my coordinate system is here, horizontal is x, vertical is y, that means my easy axis is in the x direction.

And I can actually write down what would be the matrix presentation of p0, which would present the effect of this polarizer. The p0 will be a matrix in this form, 1, 0, 0, 0. If I have this P0 acting on the z, for example, I can have the p0 acting on the z, then basically what would happen after the light passed through this polarizer will become z multiplied by p0. So basically all this p0 matrix does is to extract the position of the field in the x-axis. But I'm just writing it down in the matrix format.

And on the other hand, for example, I can have this easy axis, which is 90 degrees, which affects the x-axis, which is in the y direction, then I write this down in the form of p pi over 2. This pi over 2 is the angle between the easy axis and the x direction.

And in this case, you are going to get 0, 0, 0, 1 because all this matrix set notation does is to extract the component which is actually in the y direction. And you can of course multiply-- when you have an incident electromagnetic wave which you can extract the polarization z matrix, you just multiply p pi over 2 and z.

Basically you can actually get the resulting polarization of the electromagnetic field after passing through this polarizer. What is going to happen is that only the y component will survive. In the case of easy axis aligned in the x direction, only the x component will survive.

So now let me go through a short example here. So suppose initially I have an electromagnetic field polarized, it's linearly polarized in the y direction. And I have that electromagnetic field passing through a polarizer, which is actually theta degree away from the y-axis.

So what is going to happen is that all the components which is parallel to the axis survive. All the projected components in the axis perpendicular to the easy axis didn't make it because the electrons are going to be oscillating up and down like crazy to compensate and then refract that component.

So what is going to happen is that after passing through the easy axis, the direction of the polarization will be altered such that it's actually in line with the easy axis of the polarizer. So you can see that, after passing through this polarizer, the direction of the polarization, this is still linearly polarized, the direction is changed by theta degree.

And also the amplitude also changed because only the component parallel to the easy axis survived. Therefore, the magnitude of the E vector becomes E0 cosine of cosine theta. And therefore, the amplitude of the light got reduced. It becomes I0 cosine theta squared because the intensity of the light is proportional to E squared.

So after all this, this is time to take a look at some demonstration here. So here I have a setup. I have emission of linearly polarized microwave from the left-hand side, which is the source. And I have a receiver which is connected to this scope.

You can see the result on the scope. You can see that indeed they are energy passing from this source. And it got accepted and then recorded by the scope.

Now I have all of those-- OK, all are not so perfect, but those are metals which have many, many strips or many, many little rods here. And my initial linearly polarized wave is actually pointing in the vertical direction.

So if I have my polarizer arranged in this direction, can you predict what will happen to the readout on the scope? Will I see signal or not? How many of you think we will see signal if I arrange that the electromagnetic wave is polarized in the up and down direction? OK, one, two, three-- only four people think so.

How many of you think nothing will be observed by the scope? Most of you actually think so. So let's really do this experiment. Look at what is happening. Can you see it? Do I see a signal? No.

All the signals are canceled, right? Why? Because all the hard work of all those electrons in the metal, they are like crazy, oh my god, this is a disaster. I am going to oscillate up and down-- cancel, cancel, cancel. Then it got cancelled.

On the other hand, I can save all those electrons. So now I'm going to rotate this thing by 90 degrees and see what is going to happen. Can you see it? Nothing happened, right?

Because those electrons-- oh my god, crisis coming, but now there's nothing I can do because I cannot move up and down. Therefore, there will be no refraction. So they have just to accept the fact that this electromagnetic field went through.

All right, and that actually does something really interesting. Look at this-- so now I can actually make electromagnetic field completely destroyed, because originally the electromagnetic field is linearly polarized, up and down in this direction.

So if I have a plate which is 45 degrees and I put in another one here-- it doesn't really work very well. The signal probably is too small.

How about we do this demo in a different way. So right now here I have my computer here. Can you see my computer? You can see the screen, right? The screen essentially is made of LED.

Those LED screens emit polarized light. And those are the polarizer, which is the equivalent version of those metals but arranged in a really fine grain-- so you cannot really see all those strips. And they look pretty transparent.

But the idea is pretty similar. And the polarizing axis is in the horizontal direction. The easy axis is in horizontal direction.

So let me put this here. Can you still see the screen? You cannot, right. But if I rotate this by 90 degrees, you can see it. That is because all those light emitted from the screen it linearly polarized up and down direction.

So now if I have another identical polarizer which I insert between these two, so now you cannot see anything because all the light is pointing up and down, or the electric field is pointing up and down. And now if I insert another one, which is 45 degrees, can you see something?

You can see it-- why? That is because if I insert this additional polarizer, like what I was discussing, it's going to take the position to the direction of the easy axis.

And if I have another one, which is the easy axis pointing toward the x direction, then again since this vector is tilted already by some theta degree, then you can see some residual component which can pass through the second polarizer.

On the other hand, if I remove this polarizer in between, then what is going to happen is that all the components is pointing to the y direction. Then you cannot see any component which pass through this polarizer.

So that's actually pretty interesting. And I can rotate this, and you can see that the magnitude is changing. Only when I have 45 degrees, I see a maximum intensity. Any questions? Very good.

So now what I'm going to do is to discuss with you an interesting question, which was posted by Einstein. Einstein said that, as I said, "God doesn't play dice with the world." So that's actually what he believed.

So we can do an interesting experiment, which I have single photon source. So what do I mean by single photon source? I can emit one photon at a time-- just one, and then the second one, and the third one-- and have then pass through some imaginary polarizer.

So let's suppose I have unpolarized light with intensity at 0. So basically, after it passed through this polarizer with a axis pointing to the x direction, in this case the x direction is pointing to this direction, what you are going to get is that you are going to filter from all those unpolarized direction, you are going to filter only the light which is actually parallel to the easy axis.

So therefore, after the unpolarized light source passes through this polarizer, you are going to have all the electric field pointing to the x direction. Now everybody can accept. And of course if I have a second one, which is in the y direction, since there will be no component pointing to the y direction, you get zero electric field. That's very nice.

So Einstein was really happy that, oh, that means unpolarized light maybe is just 50% of the polarized light in the x direction and 50% of the polarized light in the y direction. Because each time I emit one photon, the first half of them got stopped by the x direction easy axis polarizer. And then the second half got stopped by the second polarizer. So that probably makes sense.

But how about-- as we did in the demonstration-- how about we rotate the second polarizer by 45 degrees? What will be the intensity? But now, hey, you cannot split a single photon because a photon is a photon. How can I split, right? Because if I describe my unpolarized light as 50% linearly polarized in x, the other half is linearly polarized in y, then I am in trouble because I don't know how to calculate what is this intensity.

So basically maybe all of them pass through or all of them doesn't pass through. Then we can do this experiment. And this is the result-- what you are going to get is that the intensity is going to be I0 over 4.

That is actually not 0 or I over 2. So that means really the nature of plate dies because a single photon is equivalent to something which can be described by a wave. So that gives us some possible connection to quantum mechanics, because that means a single photon is not like a single object, which is actually passing through all those polarizers. But they actually act also like waves.

So that is something which we'll follow up with later lectures. When we talk about interference, et cetera, we are going to also discuss related issues about the connection to quantum mechanics.

So today, we actually have learned about polarization, linearly polarized, circularly polarized, and elliptically polarized electromagnetic waves, and also unpolarized light source. And we have learned about how to produce polarized electromagnetic wave with polarizer. Basically have the unpolarized light source pass through a polarizer, you'll have a polarized light.

We didn't cover quarter wave plate yet. We are going to cover that next time. And also, next time we will talk about how we actually can generate electromagnetic field. We have been talking about electromagnetic field for a long time, but how are they actually generated is an issue which we have actually touched. And that is actually going to happen after the midterm next week.

Thank you very much. And if you have any questions, please let me know.

AUDIENCE: [INAUDIBLE]