# Massachusetts Institute of Technology

Physics 8.03 Fall 2016
Homework 6

## Problems

### Problem 6.1 (25 pts)

Consider sound waves propagating along the x-axis in an organ pipe as described by the wave equation for the longitudinal displacement, $\psi$, of a volume element of air,

$$\frac{\partial^2 \psi}{\partial t^2} = A \frac{\partial^2 \psi}{\partial x^2}$$

where the constant, $A = 90000 m^2/s^2$. The organ pipe is closed at one end, $x = 0$, and open at the other end, $x = L = 1.5$m.

a. Make a drawing of the first 3 normal modes over the interval, $0 \leq x \leq L$, (i.e., sketch $\psi$ as a function of $x$ at maximum displacement), and write down the normal mode frequencies for each. This problem does not require any elaborate calculations or solutions to above equation. All you need is just a physical understanding of how the normal modes are determined and some very rudimentary arithmetic.

b. For the first 3 normal modes, sketch the pressure as a function of $x$ at its maximum amplitude

c. If the pipe is changed to be open at both ends, how long must it be made in order to preserve the frequency of the fundamental mode?

### Problem 6.2 (25 pts)

During the lecture, we discussed the wave solution from Maxwell's equations. But we did not finish the derivation of the wave equation for magnetic field.

a. Show that $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - (\vec{\nabla} \cdot \vec{\nabla})\vec{A}$, where $\vec{A}$ is a vector.

b. Show that in vacuum, $\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$, using the Maxwell's equations.

### Problem 6.3 (25 pts)

The goal of this problem is to show that a plane pulse, a pulse traveling in some direction with no variation (at a given instant of time) prependicular to that direction is a solution to Maxwell's equations. We will choose the direction of propagation to be along the x axis:

$$\vec{E}(\vec{r}, t) = E_0 \hat{y} f(x - ct)$$

Where $f(\xi)$ is any arbitrary, well behaved function.

a. Show that this field satisfies the EM wave equation

b. Show that the field satisfies $\vec{\nabla} \cdot \vec{E} = 0$. What other choices for the vector direction of $\vec{E}$ are consistent with this Maxwell equation (Gauss's Law)?

c. Find an expression for the magnetic field associated with this pulse.

## Problem 6.4 (25 pts)

In a place far away from Earth, two sinusoidal EM plane waves, both of frequency $\nu$ and electric field amplitude $E_0$ along $\hat{y}$, travel in opposite directions in this empty space along the $\hat{x}$ direction. At $t = 0$, the electric field is found to be 0 at $x = 0$.

a. Write down the the electric field of a sinusoidal traveling wave going to the positive $x$ direction. (Hint: See lecture note 12, page 6 and the example discussed during the lecture. In general, the electric field of a progressing sinusoidal EM wave can be written as $\vec{E}(\vec{r}, t) = Re(\vec{E}_0 e^{j(\vec{k}\cdot\vec{r}-\omega t)})$, where $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ and $\hat{k} = \frac{\vec{k}}{|\vec{k}|}$ is the direction of propagation)

b. Find the total $\vec{E}(\vec{r}, t)$ of the two plane waves and the time average of $E^2(\vec{r}, t)$ (averaged over one period).

c. Find the corresponding $\vec{B}(\vec{r}, t)$ of the two plane waves and the time average of $B^2(\vec{r}, t)$ (averaged over one period).

d. Find the energy density, $U(\vec{r}, t)$ and its time average (averaged over one period).

e. Find the Poynting vector $\vec{S}(\vec{r}, t)$ and its time average (averaged over one period).