YEN-JIE LEE: So welcome, everybody. My name is Yen-Jie Lee. I am a assistant professor of physics in the physics department, and I will be your instructor of this semester on 8.03. So of course, one first question you have is, why do we want to learn about vibrations and waves? Why do we learn about this? Why do we even care?

The answer is really, simple. If you look at this slide, you can see that the reason you can follow this class is because I'm producing sound wave by oscillating the air, and you can receive those sound waves. And you can see me-- that's really pretty amazing by itself-- because there are a lot of photons or electromagnetic waves. They are bouncing around in this room, and your eye actually receive those electromagnetic waves. And that translates into your brain waves. You obviously, start to think about what this instructor is trying to tell you.

And of course, all those things we learned from 8.03 is closely connected to probability density waves, which you will learn from 8.04, quantum physics. And finally, it's also, of course, related to a recent discovery of the gravitational waves.

When we are sitting here, maybe there are already some space-time distortion already passing through our body and you don't feel it. When I'm moving around like this, I am creating also the gravitational waves, but it's so small to be detected.

So that's actually really cool. So the take-home message is that we cannot even recognize the universe without using waves and the vibrations. So that's actually why we care about this subject. And the last is actually why this subject is so cool even without quantum, without any fancy names.

So what is actually the relation of 8.03 to other class or other field of studies? It's closely related to classical mechanics, which I will use it immediately, and I hope you will still remember what you have learned from 8.01 and 8.02. Electromagnetic force is actually closely related also, and we are going to use a technique we learned from this class to understand optics, quantum mechanics, and also there are many practical applications, which you will learn from this class.

This is the concrete goal. We care about the future of our space time. We would like to predict what is going to happen when we set up an experiment. We would like to design experiments which can improve our understanding of nature. But without using the most powerful tool is very, very difficult to make progress.

So the most powerful tool we have is mathematics. You will see that it really works in this class. But the first thing we have to learn is how to translate physical situations into mathematics so that we can actually include this really wonderful tool to help us to solve problems. Once we have done that, we will start to look at single harmonic oscillator, then we try to couple all those oscillators together to see how they interact with each other.

Finally, we go to an infinite number of oscillators. Sounds scary, but it's actually not scary after all. And we will see waves because waves are actually coming from an infinite number of oscillating particles, if you think about it.

Then we would do Fourier decomposition of waves to see what we can learn about it. We learn how to put together physical systems. That brings us to the issue of boundary conditions, and we will also enjoy what we have learned by looking at the phenomenon related to electromagnetic waves and practical application and optics.

Any questions? If you have any questions, please stop me any time. So if you don't stop me, I'm going to continue talking. So that gets started.

So the first example, the concrete example I'm going to talk about is a spring block, a massive block system. So this is actually what I have on that table. So basically, I have a highly-idealized spring. This is ideal spring with spring constant, k, and the natural length L0. So that is actually what I have.

And at t equal to 0, what I am going to do is I am going to-- I should remove this mass a little bit, and I hold this mass still and release that really carefully. So that is actually the experiment, which I am going to do. And we were wondering what is going to happen afterward.

Well, the mass as you move, will it stay there or it just disappear, I don't know before I solved this question. Now I have put together a concrete question to you, but I don't know how to proceed because you say everything works. What I am going to do? I mean, I don't know.

So as I mentioned before, there is a pretty powerful tool, mathematics. So I'm going to use that, even though I don't know why mathematics can work. Have you thought about it? So let's try it and see how we can make progress.

So the first thing which you can do in order to make progress is to define a coordinate system. So here I define a coordinate system, which is in the horizontal direction. It's the x direction. And the x equal to 0, the origin, is the place which the spring is not stressed, is at its natural length. That is actually what I define as x equal to 0.

And once I define this, I can now express what is actually the initial position of the mass by these coordinates is x0. It can be expressed as x initial. And also, initially, I said that this mass is not moving. Therefore, the velocity at 0 is 0.

So now I can also formulate my question really concretely with some mathematics. Basically, you can see that at time equal to t, I was wondering where is this mass. So actually, the question is, what is actually x as a function of t?

So you can see that once I have the mathematics to help me, everything becomes pretty simple. So once I have those defined, I would like to predict what is going to happen at time equal to t. Therefore, I would like to make use physical laws to actually help me to solve this problem.

So apparently what we are going to use is Newton's law. And I am going to go through this example really slowly so that everybody is on the same page. So the first thing which I usually do is now I would like to do a force diagram analysis.

So I have this mass. This setup is on Earth, and the question is, how many forces are acting on this mass? Can anybody answer my question.

AUDIENCE: Two. We got the-- So acceleration and the spring force.

YEN-JIE LEE: OK, so your answer is two. Any different? Three. Very good. So we have two and three. And the answer actually is three. So look at this scene. I am drawing in and I have product here. So this is actually the most difficult part of the question, actually. So once you pass this step, everything is straightforward. It's just mathematics. It's not my problem any more, but the math department, they have problem, OK?

All right. So now let's look at this mass. There are three forces. The first one as you mentioned correctly is F spring. It's pulling the mass. And since we are working on Earth, we have not yet moved the whole class to the moon or somewhere else, but there would be gravitational force pointing downward. But this whole setup is on a table of friction, this table. Therefore, there will be no more force. So don't forget this one. There will be no more force. So the answer is that we have three forces.

The normal force is, actually, a complicated subject, which you will need to understand that will quantum physics. So now I have three force, and now I can actually calculate the total force, the total force, F. F is equal to Fs plus Fn plus Fg.

So since we know that the mass is moving in the horizontal direction, the mass didn't suddenly jump and disappear. So it is there. Therefore, we know that the normal force is actually equal to minus Fg, which is actually Ng in the y direction. And here I define y is actually pointing up, and the x is pointing to the right-hand side.

Therefore, what is going to happen is that the total force is actually just Fs. And this is equal to minus k, which is the spring constant and x, which is the position of the little mass at time equal to t. So once we have those forces and the total force, actually, we can use Newton's law.

So F is equal to m times a. And this is actually equal to m d squared xt dt squared in the x direction, and that is actually equal to mx double dot t x. So here is my notation. I'm going to use each of the dot is actually the differentiation with respect to t. So this is actually equal to minus kxt in the x direction.

So you can see that here is actually what you already know about Newton's law. And that is actually coming from the force analysis. So in this example, it's simple enough such that you can write it down immediately, but in the later examples, things will become very complicated and things will be slightly more difficult. Therefore, you will really need help from the force diagram.

So now we have everything in the x direction, therefore, I can drop the x hat. Therefore, finally, my equation of motion is x double dot t. And this is equal to minus k over n x of t.

To make my life easier, I am going to define omega equal to square root of k over n. You will see why afterward. It looks really weird why professor Lee wants to do this, but afterward, you will see that omega really have a meaning, and that is equal to minus omega squared x.

So we have solved this problem, actually, as a physicist. Now the problem is what is actually the solution to this differential second-order differential equation. And as I mentioned, this is actually not the content of 8.03, actually, it's a content of 18.03, maybe. How many of you actually have taken 18.03?

Everybody knows the solution, so very good. I am safe. So what is the solution? The solution is x of t equal to a cosine of omega t plus b sine omega t. So my friends from the math department tell me secretly that this is actually the solution. And I trust him or her.

So that's very nice. Now I have the solution, and how do I know this is the only solution? How do I know? Actually, there are two unknowns, just to remind you what you have learned. There are two unknowns. And if you plug this thing into this equation, you satisfy that equation. If you don't trust me, you can do it offline. It's always good to check to make sure I didn't make a mistake. But that's very good news.

So that means we will have two unknowns, and those will satisfy the equation. So by uniqueness theorem, this is actually the one and the only one solution in my universe, also yours, which satisfy the equation because of the uniqueness theorem. So I hope I have convinced you that we have solved this equation.

So now I take my physicist hat back and now it is actually my job again. So now we have the solution, and we need to determine what is actually these two unknown coefficients. So what I'm going to use is to use the two initial conditions.

The first initial condition is x of 0 equal to x initial. The second one is that since I released this mass really carefully and the initial velocity is 0, therefore, I have x dot 0 equal to 0. From this, you can solve. Plug these two conditions into this equation. You can actually figure out that a is equal to x initial. And b is equal to 0.

Any questions so far? Very good. So now we have the solution. So finally, what is actually the solution? The solution we get is x of t equal to x initial cosine omega t.

So this is actually the amplitude of the oscillation, and this is actually the angular velocity. So you may be asking why angular? Where is the angular coming from? Because this is actually a one-dimensional motion. Where is the angular velocity coming from? And I will explain that in the later lecture.

And also this is actually a harmonic oscillation. So what we are actually predicting is that this mass is going to do this, have a fixed amplitude and it's actually going to go back and forth with the angular frequency of omega. So we can now do an experiment to verify if this is actually really the case.

So there's a small difference. There's another spring here, but essentially, the solution will be very similar. You may get this in a p-set or exam.

So now I can turn on the air so that I make this surface frictionless. And you can see that now I actually move this thing slightly away from the equilibrium position, and I release that carefully.

So you can see that really it's actually going back and forth harmonically. I can change the amplitude and see what will happen. The amplitude is becoming bigger, and you can see that the oscillation amplitude really depends on where you put that initially with respect to the equilibrium position. I can actually make a small amplitude oscillation also. Now you can see that now the amplitude is small but still oscillating back and forth.

So that's very encouraging. Let's take another example, which I actually rotate the whole thing by 90 degrees. You are going to get a question about this system in your p-set. The amazing thing is that the solution is the same. What is that?

And you don't believe me, let me do the experiment. I actually shifted the position. I changed the position, and I release that really carefully. You see that this mass is oscillating up and down. The amplitude did not change. The frequency did not change as a function of time.

It really matched with the solution we found here. It's truly amazing. No? The problem is that we are so used to this already. You have seen this maybe 100 times before my lecture, so therefore, you got so used to this.

Therefore, when I say, OK, I make a prediction. This is what happened, you are just so used to this or you don't feel the excitement. But for me, after I teach this class so many times, I still find this thing really amazing.

Why is that? This means that actually, mathematics really works, first of all. That means we can use the same tool for the understanding of gravitational waves, for the prediction of the Higgs boson, for the calculation of the property of the quark-gluon plasma in the early universe, and also at the same time the motion of this spring-mass system.

We actually use always the same tool, the mathematics, to understand this system. And nobody will understands why. If you understand why, please tell me. I would like to know. I will be very proud of you.

Rene Descartes said once, "But in my opinion, all things in nature occur mathematically." Apparently, he's right. Albert Einstein also once said, "The most incomprehensible thing about the universe is that it is comprehensible." So I would say this is really something we need to appreciate the need to think about why this is the case. Any questions?

So you may say, oh, come on. We just solved the problem of an ideal spring. Who cares? It's so simple, so easy, and you are making really a big thing out of this. But actually, what we have been solving is really much more than that. This equation is much more than just a spring-mass system.

Actually, if you think about this question carefully, there's really no Hooke's law forever. Hooke's law will give you a potential proportional to x squared. And if you are so far away, you pull the spring so really hard, you can store the energy of the whole universe. Does that make sense? No.

At some point, it should break down. So there's really no Hook's law. But there's also Hook's law everywhere. If you look at this system, it follows the harmonic oscillation.

If you look at this system I perturb this, it goes back and forth. It's almost like everywhere. Why is this the case? I'm going to answer this question immediately.

So let's take a look at an example. So if I consider a potential, this is an artificial potential, which you can find in Georgi's book, so v is equal to E times L over x plus x over L. And if you practice as a function of x, then basically you get this funny shape. It's not proportional to x squared. Therefore, you will see that, OK, the resulting motion for the particle in this potential, it's not going to be harmonic motion.

But if I zoom in, zoom in, and zoom in and basically, you will see that if I am patient enough, I zoom in enough, you'll see that this is a parabola. Again, you follow Hooke's law. So that is actually really cool. So if I consider an arbitrary v of x, we can do a Taylor expansion to this potential.

So basically v of x will be equal to v of 0 plus v prime 0 divided by 1 factorial times x plus v double prime 0 over 2 factorial x squared plus v triple prime 0 divided by 3 factorial x to the third plus infinite number of terms. v 0 is the position of where you have minimum potential.

So that's actually where the equilibrium position is in my coordinate system. It's the standard, the coordinate system I used for the solving the spring-mass question. So if I calculate the force, the force, f of x, will be equal to minus d dx v of x. And that will be equal to minus v prime 0 minus v double prime 0 x minus 1 over 2 v triple prime 0 x squared plus many other terms.

Since I have mentioned that v of 0-- this will be x. v of 0 is actually the position of the minima. Therefore, v prime of 0 will be equal to 0. Therefore. This term is gone. So what essentially is left over is the remaining terms here.

Now, if I assume that x is very small, what is going to happen? Anybody know when x is very small, what is going to happen? Anybody have the answer?

AUDIENCE: [INAUDIBLE].

YEN-JIE LEE: Exactly. So when x is very small, he said that the higher order terms all become negligible. OK? So that is essentially correct. So when x is very small, then I only need to consider the leading order term. But how small is the question. How small is small?

Actually, what you can do is to take the ratio between these two terms. So if you take the ratio, then basically you would get a condition xv triple dot 0, which will be much smaller than v double prime 0. So that is essentially the condition which is required to satisfy it so that we actually can ignore all the higher-order terms.

Then the whole question becomes f of x equal to minus v double prime 0 x. And that essentially, Hooke's law. So you can see that first of all, there's no Hooke's law in general. Secondly, Hook's law essentially applicable almost everywhere when you have a well-behaved potential and if you only perturb the system really slightly with very small amplitude, then it always works.

So what I would like to say is that after we have done this exercise, you will see that, actually, we have solved all the possible systems, which have a well-behaved potential. It has a minima, and if I have the amplitude small enough, then the system is going to do simple harmonic oscillation. Any questions? No question, then we'll continue.

So let's come back to this equation of motion. x double dot plus omega squared x, this is equal to 0. There are two important properties of this linear equation of motion. The first one is that if x1 of t and x2 of t are both solutions, then x12, which is the superposition of the first and second solution, is also a solution.

The second thing, which is very interesting about this equation of motion, is that there's a time translation invariance. So this means that if x of t is a solution, then xt prime equal to xt plus a is also a solution. So that is really cool, because that means if I change t equal to 0, so I shift the 0-th time, the whole physics did not change.

So this is actually because of the chain law. So if you have chain law dx t plus a dt, that is equal to d t plus a dt, dx t prime dt prime evaluated at t prime equal to t plus a. And that is equal to dx t prime dt, t prime equal to t plus a.

So that means if I have changed the t equal to 0 to other place, the whole equation of motion is still the same. On the other hand, if the k, or say the potential, is time dependent, then that may break this symmetry. Any questions?

So before we take a five minute break, I would like to discuss further about this point, this linear and nonlinear event. So you can see that the force is actually linearly dependent on x. But what will happen if I increase x more? Something will happen. That means the higher-ordered term should also be taken into account carefully.

So that means the solution of this kind, x initial cosine omega t, will not work perfectly. In 8.03, we only consider the linear term most of the time. But actually, I would like to make sure that everybody can at this point, the higher-order contribution is actually visible in our daily life.

So let me actually give you a concrete example. So here I have two pendulums. So I can now perturb this pendulum slightly. And you you'll see that it goes back and forth and following simple harmonic emotion.

So if I have both things slightly oscillating with small amplitude, what is going to happen is that both pendulums reach maxima amplitude at the same time. You can see that very clearly. I don't need to do this carefully.

You see that they always reach maxima at the same time when the amplitude is small. Why? That is because the higher-order terms are not important.

So now let's do a experiment. And now I go crazy. I make the amplitude very large so that I break that approximation. So let's see what will happen. So now I do this then. I release at the same time and see what will happen. You see that originally they are in phase. They are reaching maxima at the same time.

But if we are patient enough, you see that now? They are is oscillating, actually, at different frequencies. Originally, the solution, the omega, is really independent of the amplitude. So they should, actually, be isolating at the same frequency. But clearly you can see here, when you increase the amplitude, then you need to consider also the nonlinear effects.

So any questions before we take a five-minute break. So if not, then we would take a five-minute break, and we come back at 25.

So welcome back, everybody. So we will continue the discussion of this equation of motion, x double dot plus omega square x equal to 0. So there are three possible way to like the solution to this equation. So the first one as I mentioned before, x of t equal to a cosine omega t plus b sine omega t.

So this is actually the functional form we have been using before. And we can actually also rewrite it in a different way. So x or t equal to capital A cosine omega t plus phi. You may say, wait a second. You just promised me that this is the first one, the one is the one and only one solution in the universe, which actually satisfy the equation of motion.

Now you write another one. What is going on? Why? But actually, they are the same. This is actually A cosine phi cosine omega t minus A sine phi sine omega t.

So the good thing is that A and phi are arbitrary constant so that it should be you can use two initial conditions to determine the arbitrary constant. So you can see that one and two are completely equivalent. So I hope that solves some of the questions because you really find it confusing why we have different presentations of the solution.

So there's a third one, which is actually much more fancier. The third one is that I have x of t. This is actually a real part of A-- again, the amplitude-- exponential i omega t plus phi, where i is equal to the square root of minus 1.

Wait a second. We will say, well, professor, why are you writing such a horrible solution? Right? Really strange. But that will explain you why. So three is actually a mathematical trick. I'm not going to prove anything here because I'm a physicist, but I would like to share with you what I think is going on.

I think three is really just a mathematical trick from the math department. In principle, I can drive it an even more horrible way. x of t equal to a real part of A cosine omega t plus phi plus i f of t. And f of t is a real function. In principle, I can do that. It's even more horrible. Why is that? Because I now have this function. I take the real part, and I actually take the two out of this operation.

So f of t is actually the real function. It can be something arbitrary. And i can now plot the locus of this function, the solution on the complex print. Now I'm plotting this solution on this complex print. What is going to happen is that you're going to have-- That is what you are going to get.

If I am lucky, if this f of t is confined in some specific region, if I not lucky, then it goes out of the print there. I couldn't see it. Maybe it go to the moon or something. But if you are smart enough, and I'm sure you are, if I choose f of t equal to A sine omega t plus phi, can anybody tell me what is going to happen?

AUDIENCE: [INAUDIBLE].

YEN-JIE LEE: Would you count a circle? Very good. If I plot the locus again of this function, the real axis, imaginary axis, then you should get a circle. Some miracle happened. If you choose the f of t correctly, wisely, then you can actually turn all this mess into order. Any questions?

So I can now follow up about this. So now I have x of t is equal to the real part of A cosine omega t plus phi plus iA sine omega t plus phi. And just a reminder, exponential i theta is equal to cosine theta plus i sine theta. Therefore, I arrive this. This is a real part of A exponential i omega t plus phi.

So if I do this really carefully, I look at this the position of the point at a specific time. So now time is equal to t. And this is the real axis, and this is the imaginary axis. So I have this circle here.

So at time equal to t, what you are getting is that x is actually-- before taking the real part, A, exponential i omega t plus phi, it's actually here. And this vector actually shows the amplitude. Amplitude is A. And the angle between this vector pointing to the position of this function is omega t plus phi. So this is actually the angle between this vector and the real axis.

So that's pretty cool. Why? Because now I understand why I call this omega angular velocity or angular frequency. Because the solution to the equation of motion, which we have actually derived before, is actually the real part of rotation in a complex print.

If you think about it, that means now I see this particle going up and down. I see this particle going up and down. You can think about that, this is Earth. If there is an extra dimension mention, which you couldn't see. Actually, this particle in the dimension where we can see into the extra dimension, which is hidden is actually rotating.

And while we see that reality, it's a projection to the real axis. You see? So in reality, this particle is actually rotating, if you add the image and the extra dimension. So that is actually pretty cool, but the purity artificial. So you can see that I can choose f of t to be a different function, and then this whole picture is different. But I also would create a lot of trouble because then the mathematics become complicated. I didn't gain anything.

But by choosing this functional form, you actually write a very beautiful picture.

Another thing, which is very cool about this is that if I write this thing in the exponential functional form, since we are dealing with differential equations, there is a very good property about exponential function. That is it is essentially a phoenix function. Do you know what is a phoenix? Phoenix is actually some kind of animal, a long-beaked bird, which is cyclically called the regenerated or reborn.

So basically, when this phoenix die, you will lay the eggs in the fire and you were reborn. This is actually the same as this function. I can do differentiation, still an exponential function, and differentiate, differentiate, differentiate. Still exponential function. So that is very nice because when we deal with differential equation, then you can actually remove all those dots and make them become just exponential function. So essentially, a very nice property.

So the first property, which is very nice is that it cannot be killed by differentiation. You will see how useful this is in the following lectures. The second thing, which is really nice is that it has a very nice property. So basically the exponential i theta 1 times exponential i theta 2, and that will give you exponential i theta 1 plus theta 2.

So what does that mean? That means if I have a solution in this form, A exponential i omega t plus phi. And I do a times translation, t become t plus A. Then this become A exponential i omega t plus A plus phi.

So this means that times translation in this rotation is just a rotation in complex print. You see?

So now t becomes t plus A. Then you are actually just changing the angle between this vector and the x-axis. So as time goes on, what is going to happen is that this thing will go around and around and around and the physics is always the set, no matter when you start counting, and the translation is just the rotation in this print. Any questions?

So I think this is actually a basic slide just to remind you about Euler's formula. So basically, the explanation i phi is equal to cosine phi plus i sine phi. And I think it will be useful if you are not familiar with this. It is useful to actually review a little bit about exponential function, which will be very useful for this class.

So I'm running a bit faster today. So let's take a look at what we have learned today. We have analyzed the physics of a harmonic oscillator. So basically, we start by asking really just a verbal question, what is going to happen to this mass on the table attached to a spring.

And what we have learned is that we actually use mathematics. Basically, we translate all what we have learned about this mass into mathematics by first define a coordinate system. Then I'd write everything using that coordinate system.

Then I use Newton's law to help us to solve this question. And we have analyzed the physics of this harmonic oscillator. And Hooke's law, we found that he actually, not only works for this spring-mass system, it also works for all kinds of different small oscillations about a point of equilibrium. So basically, it's actually a universal solution what we have been doing.

And we have found out a complex exponential function is actually a beautiful way to present the solution to the equation of motion we have been studying. So everything is nice and good. However, life is hard because there are many things which actually, we ignored in this example.

One apparent thing, which we actually ignore, is the direct force. So you can see that before I was actually making this pendulum oscillate back and forth. What is happening now? There are not oscillating anymore. Why? Well, they stopped being.

Apparently, something is missing. When I actually moved this system, if I turn off the air so that there's friction, then it doesn't really move. If I increase a bit, the air so that the slide have some slight freedom, then actually, you can see that you move a bit then you stop. If I increase this some more, you can see that the amplitude becomes smaller and smaller.

So in the following lecture, what we are going to do is to study how to actually include a direct force into it again and of course, using the same machinery which we have learned from here and see if we can actually solve this problem.

Thank you very much. We actually end up earlier today. Sorry for that. And maybe I will make the lecture longer next time. And if you have any questions about what we have covered today, I'm here available to help you.