MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

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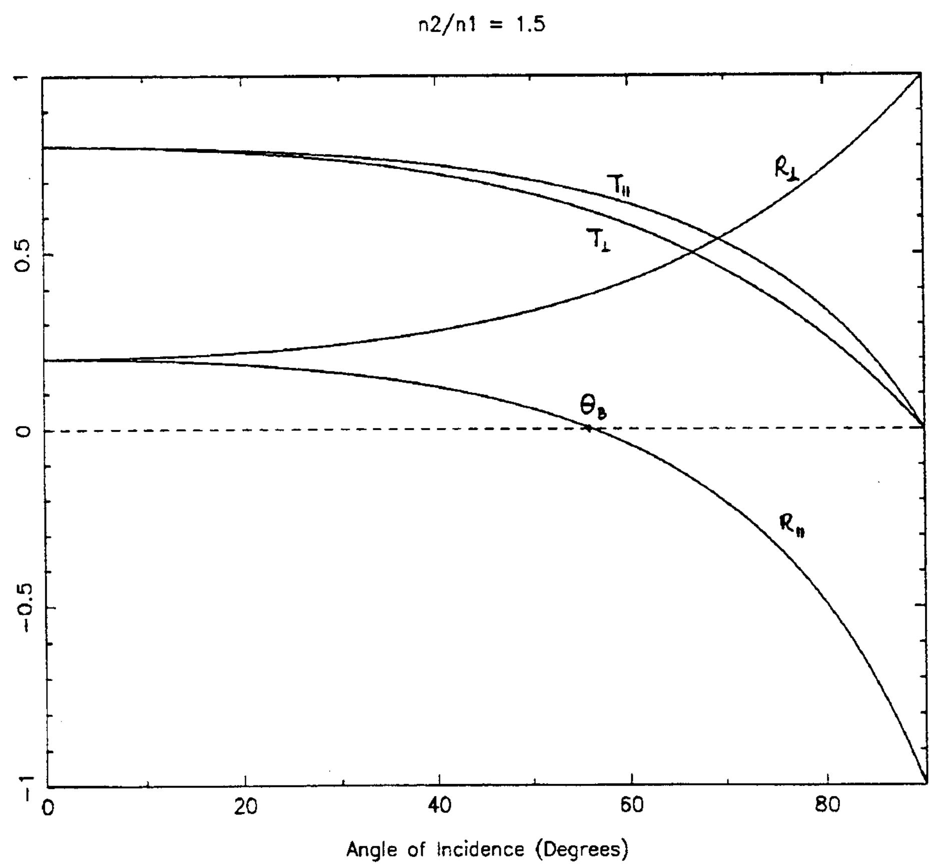
Take-Home Experiment #5

REFLECTION OF LIGHT: THE FRESNEL EQUATIONS

Objective The Fresnel equations describe the ratio of transmitted or reflected electric field to the incident electric field when electromagnetic radiation impinges on an interface between two different indices of refraction. The result depends on whether the incident electric field is parallel or perpendicular to the plane of incidence; thus there are four separate equations. The results are plotted as a function of the angle of incidence in the figure below. We will explore the physical meaning of these results by doing a simple experiment.



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Experiment Clean one of the microscope slides. Find a room with a high ceiling and a ceiling light under which you can stand comfortably. Stand erect with a hand holding the slide at your side. With your arm pointing toward the floor use the slide as a mirror to view the ceiling light. You are looking at light reflected at near normal incidence ($\theta = 0$). Actually, you are looking at two images of the light, one reflected from the top surface of the slide and one from the bottom surface. Since the two surfaces are probably not exactly parallel to each other, you may be able to distinguish two different images of some sharp feature on the light fixture.

Note in the figure that at $\theta = 0$, $R_{||}$ and $R_{\perp}$ have identical magnitudes of 0.2. Therefore each image has 4% of the incident intensity and the combined intensity is 8%. You should have no difficulty perceiving that the reflected image is weaker than that seen when looking directly at the ceiling light. You may wish to compare this reflection to that from a small mirror held in a similar position.

Slowly raise your arm in a circular arc in front of you. Simultaneously tilt the slide so that the ceiling light remains in view. You should see that the image reflected by the slide is getting brighter. When your arm is pointing almost at the light the image you see is reflected at grazing incidence; that is, θ is almost 90 degrees. In this position you should be able to compare the reflected and the direct image of the light. You should see that there is no apparent difference in the brightness of the two images. This corresponds to the right hand side of the graph where the magnitude of the electric field ratio for reflection goes to one. The clear glass slide becomes a perfect mirror near grazing incidence!

Repeat the above experiment using a polarizer near your eye. The Fresnel equations predict that at normal incidence and at grazing incidence the reflectivity will be independent of polarization. Test this first. Now sweep through the full range of angles with the easy axis of the polarizer horizontal. This corresponds to $R_{\perp}$ in the graph. The brightness of the reflected image should be a monotonically increasing function of θ. Next hold the easy axis of the polarizer vertically. How does the brightness vary with angle now? Can you tell when you are at Brewster's angle?

You can try to measure Brewster's angle as follows. Determine a position on a wall of the room that is exactly at your shoulder height. While holding the slide directly under the light, find Brewster's angle and then sight through the slide to a second position on the wall above the first. Measure the difference, h, between these two heights and the distance, d, of your feet from the wall. Prove to yourself that under these circumstances $\theta_B = \frac{1}{2}[90° + \tan^{-1}(h/d)]$. See what value you get.

Note: If you do not have easy access to a room with a high ceiling, you could try this experiment in a horizontal plane in your own room. Position a light source at eye level. You could use the mini-maglite as a source, but it is hard to judge the intensity of a bright small source. You can get around this problem by putting a handkerchief over the face of the flashlight.

5A- LIQUID PRISM

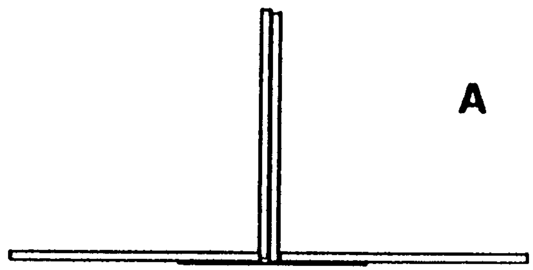
Objective In this experiment you will construct a hollow prism which can be filled with various liquids and used to study refraction.

Construction Note: The actual time necessary to make this prism is not very long; however, since the RTV requires up to 24 hours to set properly you should allow several days to complete the construction. Begin by cleaning four microscope slides. Place a piece of tape along one of the long sides of a slide so that about half the width of the tape is stuck to the face of the slide. Place the slide on a piece of notebook paper (to avoid a mess) with the tape next to the paper, sticky side up. See Figure A. Use two other slides as spacers as you stick a second slide to the tape, parallel to the first but separated from it by the two-slide-width gap. Be careful to make sure that the short edges of the two slides are aligned. Once the second slide is positioned and stuck down, you can remove the two spacer slides. Repeat this process to tape a third slide to the assembly, as shown in Figure B. Using a fine nozzle on the RTV tube, fill each of the two gaps between the taped slides with sealant.

Prepare several 3 inch lengths of tape and stick them by a small corner to some convenient spot within easy reach. Fold the three slides together so that they form an equilateral prism as shown in Figure C. Position the untaped edges so that they either just touch or are separated by a gap of about half of a millimeter. Make sure the long edges are parallel and that the short edges meet properly. You can maintain this alignment by putting tape across each open end of the prism (it does not adhere very well to the thin edges of the slides) and then folding the tape across the flat sides (where it does adhere well). Take the extra time to insure that the alignment is good and the tape has stabilized the structure. Put RTV along this third long edge of the prism. Make sure that the bead of sealant wets the entire exposed long edge of each slide. I have found that this is the most likely of the three edges to leak. Put your assembly aside and let the sealant set for about a day.

Remove the tape from the two long edges of the prism and inspect your work. Note any spots that might need touching up with more sealant. Put RTV on the the three short sides at one end of the prism. Stick it down on a fourth slide as shown in Figure D. Put an extra bead of RTV around the base of the prism and touch up any spots that need it on the long edges. Put your completed assembly away for another day to set.

Fill your prism with water. Note where the leaks are occurring. Empty and dry the prism. Patch the leaks and put the prism aside again. Repeat until the prism is leak free.



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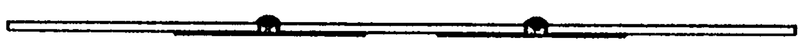
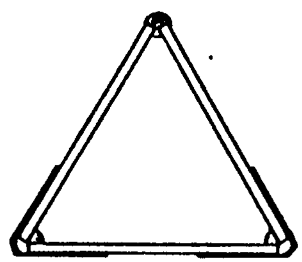
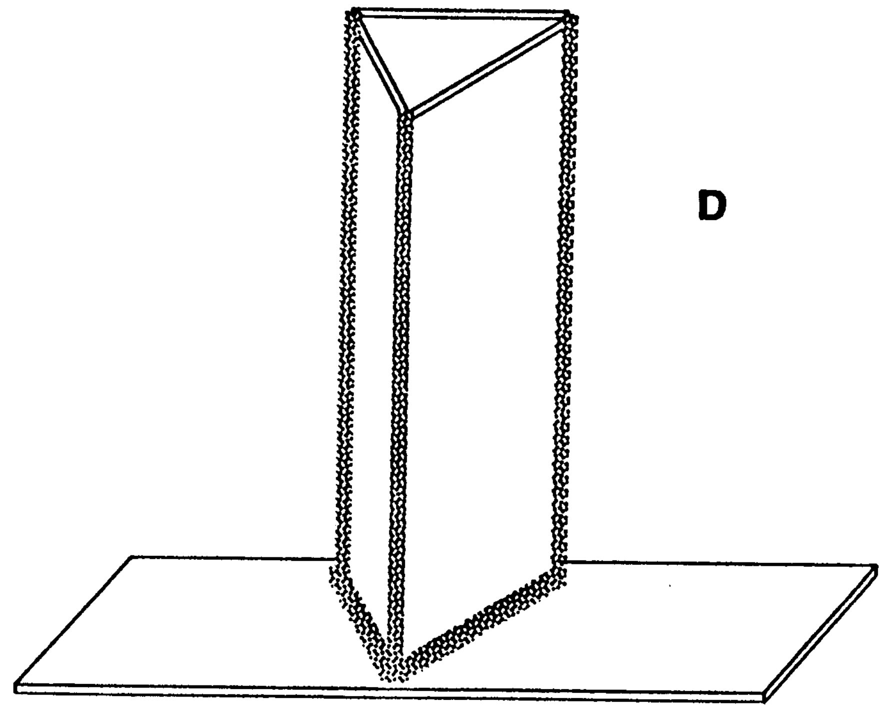


B



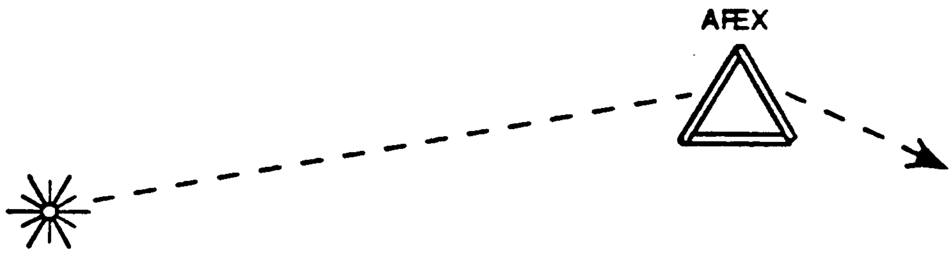
C



D

5A-2

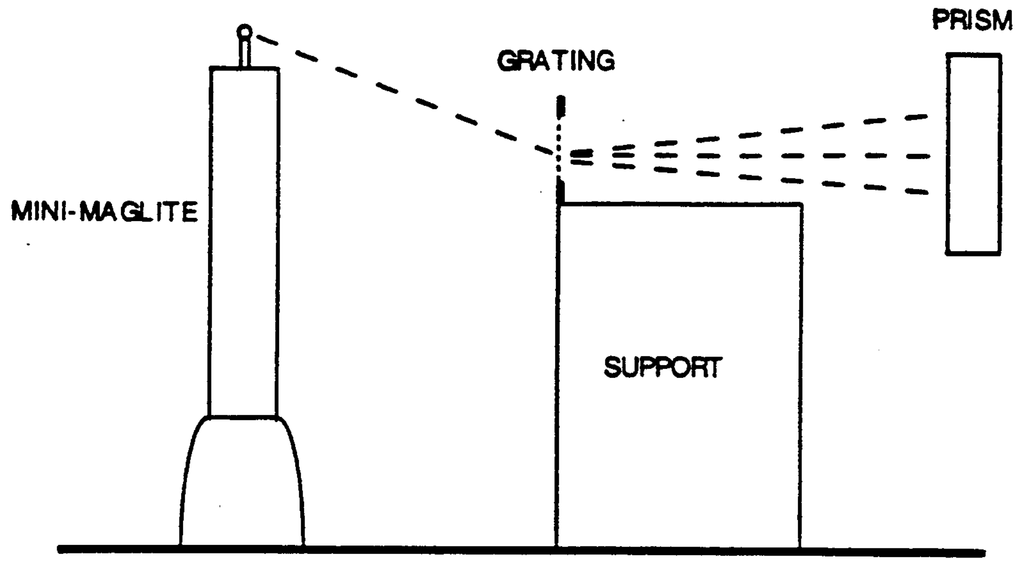
Experiments Fill the prism with water. Using the geometry shown below, view distant objects through the prism. The colors will be most pronounced around the edge of bright objects. A clean spectrum can be obtained by using the mini-maglite (with its reflector removed) as a source.



The amount the prism bends a light ray away from a straight path increases with increasing index of refraction of the liquid. It is also known that for normal materials, the index of refraction is an increasing function of the frequency of the light. Hence, colors at the blue end of the spectrum will be bent (refracted) further than colors at the red end. It is interesting to note that this is just the opposite to what happens with a diffraction grating. With a grating, the lowest frequency light gets deflected the most. But look carefully at the dispersed image of the mini-maglite bulb. The blue end of the spectrum is closer to the apex of the prism! You do not fully understand what is happening in the experiment until you can explain this apparent contradiction.

Place the prism on a sunny window sill. If you can get the sun to shine directly on the prism, you will be rewarded with a fine spectrum somewhere in your room. Now where does the blue end of the spectrum fall?

Not only does blue light have a higher index of refraction than red in normal materials, but the rate of change of the index with frequency is greater at that end of the spectrum. You can see this for yourself with the set up shown on the next page. Take the reflector off of the mini-maglite and use it as a stand. Find the transmission grating and tape it to the edge of a convenient support so that its center is about two inches lower than the bulb. Make sure the grating is oriented to disperse the light in the vertical plane. With your eye where the prism is in the figure, adjust the separation between the support and the mini-maglite so that the center of the spectrum travels horizontally across the room. Position yourself so that the spectrum appears as a colored vertical line. (By the way, which color appears highest? Do you now understand why?) Now view the dispersed spectrum through the prism. If $dn/d\lambda$ were roughly constant, the spectrum would now simply appear tilted at some angle to the vertical. Instead, it has a pronounced curvature which increases toward the blue end.



Different liquids have different dispersing power, which is not the same as saying that they have a higher average index of refraction. You should try different liquids in your prism. (Yes, I know, but beer is mainly water anyway.) One liquid which is clearly more dispersive than water is mineral oil. It can be found in most drug stores, usually in the laxative section. It makes a nice liquid to use if you plan to leave your prism on the window for the pretty effect it produces. The mineral oil does not evaporate as fast as water does, but it is a mess to clean up if it spills.

5A-4

5B - MOIRE PATTERN SIMULATION OF INTERFERENCE PHENOMENA

Objective The moiré patterns produced by overlapping sets of equally spaced concentric circles are used to demonstrate several important concepts associated with interference phenomena.

Experiment You should have three identical transparencies, each consisting of concentric circles of equal width, alternating between clear and black. Check to see that the transparencies are nearly identical by superposing one over another. The resulting pattern should look just like that of a single transparency. In this situation, the two transparencies are in-phase everywhere: clear overlapping clear, black overlapping black. Displace one transparency with respect to the other. Now there will be some regions in the resulting pattern where the two transparencies are still in-phase and the local pattern is that of alternating clear and black lines. But there will also be regions where the two transparencies are out-of-phase (clear overlapping black) and the local pattern is almost all black.

Each transparency can be used to represent a monochromatic wave spreading from a point source in two dimensions. The wavelength is the spacing between neighboring black lines; the phase of each wave at its source is the same. When waves are in-phase in some region they interfere constructively, resulting in an intensity maximum. Thus the in-phase condition pointed out above for the moiré pattern corresponds to peaks in the interference pattern of the associated waves. Waves exhibit destructive interference in regions where they are out-of-phase; in these regions the intensity pattern would have a minimum (actually a zero if the amplitudes were equal). Therefore the out-of-phase condition described above for the moiré pattern corresponds to minima in the interference pattern. You can now follow the loci of these interference maxima and minima as the sources are moved relative to each other.

Displace one transparency a number of "wavelengths" relative to a second. Observe that the loci of individual maxima and minima form hyperbolas. There is always an interference maximum along the perpendicular bisector of the line joining the two sources. We learned in class that the number of interference maxima for a geometry such as this is the integer part of $1 + (2d/\lambda)$ where d is the separation between the two sources. Verify this relation for several source separations of the two transparencies. [Suggestion: d/λ is most easily determined by looking at the relative shift of the transparencies at their largest (last) black circle.]

The location of an interference maximum can be changed by changing the relative phase with which the sources are driven, even if the source locations remain fixed. This is the principle behind phased array radar, where the radar beam can be swept in direction without changing the physical orientation of the antenna. Obviously we can not change the phase of the "sources" imprinted on the transparencies. However, we can simulate the effect at large distances from the sources by moving one of the sources perpendicular to the line joining them.

Separate two transparencies by a wavelength or two. This is most easily done by beginning with exact overlap and then displacing one relative to the other along the long axis of the the sheets. Now hold this displacement fixed while displacing the sheets by small amounts along the short axis of the sheets. You should be able to sweep smoothly the angle that the principal interference maximum makes with the center of the "array". For equal transverse displacements (equal changes in relative source phase), how does the angular excursion depend on the source separation d? In other words, is the rate of change of angle with respect to phase a function of d?

Next try using three sources. This takes careful alignment of the transparencies, but the results are worth the effort. Displace two transparencies by some amount. A source separation $d = 4\lambda$ works well. Again count line displacement at the edge of the pattern to establish the shift. Now overlay a third transparency so that it has the same displacement (in the same direction) relative to the second. It will be displaced by 2d relative to the first transparency. Hold down all three at one edge while alternately lifting and returning the third transparency. Watch the interference pattern as you change from two to three sources. You should see that the principal maxima remain in the same location but become narrower. At the same time, secondary maxima appear between the principal maxima.

In the above experiment you changed the number of sources while keeping their separation, d, constant. Next, keep the width of the array constant while changing the number of sources. Remove the top transparency and replace it so that its center is located half way between the centers of the first two transparencies. Again study changes in the moiré pattern while folding back and returning the third transparency. You should see that the number of interference maxima remains the same, but the principal maxima become more widely spaced.