MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

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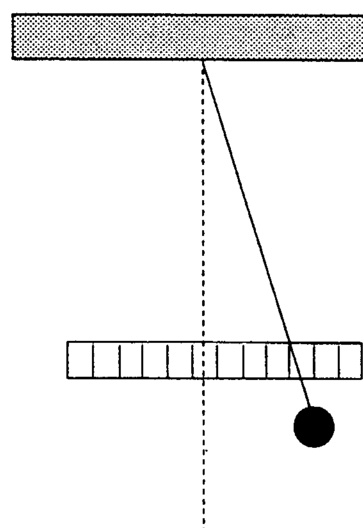
Take-Home Experiment #1

INFLUENCE OF MASS ON THE DAMPING OF A PENDULUM

Objective The period of a simple pendulum is independent of its mass. The principal damping mechanism is the resistance that air offers to the motion of the swinging object. That resistance depends on the shape of the object, but not its mass. Hence, one might expect that the decay time of the pendulum would also be independent of its mass. You will discover here that this is not the case. You will also learn why.

Experiment Use the 10 ounce weight and the braided fishing line to construct a pendulum of length about 1 to 1.5 meters. Make sure that the suspension point is rigid, so that its motion does not provide a source of damping. Slice the foam sphere in half and scoop out a cavity inside just large enough to enclose the weight. Put the sphere around the weight and secure it with a band of Scotch tape around the equator.



Find a way of mounting the plastic ruler behind the pendulum so that it can be used to indicate the deflection of the string (it is harder to measure the deflection of the bottom or the edge of the sphere). Release the pendulum from an initial amplitude of about 15 cm. Measure the period. You may try a number of successive measurements of the time it takes to go through 20 oscillation to estimate how accurately you can measure the period. You do not need an accurate measurement for this experiment, but it is an interesting exercise. Now the important measurement. Determine how long it takes the pendulum to change its amplitude by a factor of one half.

Now change weights and carry out the same measurements. Use one of the 1.5 oz weights this time. Thread the line through the hole and jam it in place with a wooden peg. This way the length of the pendulum can be made quite close to that used previously. Replace the sphere, using some cotton or tissue paper to fill the rest of the cavity so that the weight does not move relative to the sphere. Now the damping time should be noticeably shorter than was the case with the heavier weight.

Discussion The force on the object due to air friction is modeled as $F_{\text{friction}} = -b\dot{x}$. The constant b depends on such factors as the geometry of the object and the viscosity of air. It does not depend on the mass of the object. For small amplitude oscillations, $F_{\text{restoring}} = -mg(x/\ell)$. Putting these forces into the equation of motion gives

$$F_{\text{total}} = -mg(x/\ell) - b\dot{x} = m\ddot{x}.$$

Recall that the damping time is $\tau = 1/\gamma = m/b$. Thus, the damping time is proportional to the mass.

If the above explanation satisfied you, perhaps you should consider majoring in math instead of physics. Let's try again. If one moves an object a distance dx against an applied force F, an energy $dE = -Fdx$ is transferred to the system supplying the force. In the case of the damping of the pendulum, $F = -b\dot{x}$ and dE is supplied to the air in the form of an increase in the kinetic energy of the molecules. Since energy is conserved, the pendulum's energy decreases by an amount $dE = b\dot{x}dx$. The energy lost per cycle, ΔE, is the integral of this quantity over one cycle of the pendulum. One does not have to do the integral analytically to understand the result. The path for dx is uniquely determined by the maximum amplitude of the swing, x_{max}. The velocity, $\dot{x}$, is uniquely determined for the entire path if one knows, in addition to x_{max}, the frequency ω. Thus for all oscillations with the same x_{max} and ω, ΔE will be the same. This energy loss per cycle does not depend on the mass.

Now consider the average total energy, E_{total}, that the pendulum has when its maximum excursion is x_{max}. This is easiest to determine at the end of the swing when all the energy is potential and is equal to mgh. Notice that this quantity is linearly dependent on mass. The height, h, above the stationary position depends only on x_{max} and ℓ: $h \approx \frac{1}{2}x_{\text{max}}^2/\ell$. To summarize, the energy lost per cycle is independent of the mass; the average energy during that cycle is directly proportional to the mass. Thus, all other things being equal, the damping rate, which is proportional to $\Delta E/E_{\text{total}}$, is inversely proportional to the mass. τ would then be proportional to the mass.

Questions

1. What is the relation between τ discussed above and the time you measured, $t_{1/2}$, that it takes for the amplitude to decrease by a factor of 2? (Answer: $t_{1/2} = (2\ln 2)\tau$)

2. How is an infinitesimal fractional change in the period related to an infinitesimal fractional change in the length? (Answer: $dT/T = \frac{1}{2} d\ell/\ell$)

3. How well, fractionally, could you determine the period T? How accurately, in cm, would you have to reposition the mass to set T to the same value within your experimental ability to determine T?

4. What was the Q of the oscillator in each case?

5. Damping causes the actual frequency ω_f to fall below the undamped frequency ω_o. Should you be able to measure this frequency (or period) shift when you change to the lighter mass?

6. Were your damping times proportional to the mass? If they were not, do you think the discrepancy is due to the measurement accuracy or to other sources of damping?