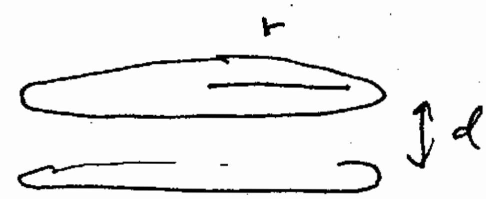
Experiment MF Analysis:

I ran 3 trials with current running in opposite directions producing a repulsive force between the magnets. From the lab, we approximate this force (see problem 6) by

$$F_{10\,coil} = \frac{\mu_0 n_1 n_2 i^2 r}{d}$$

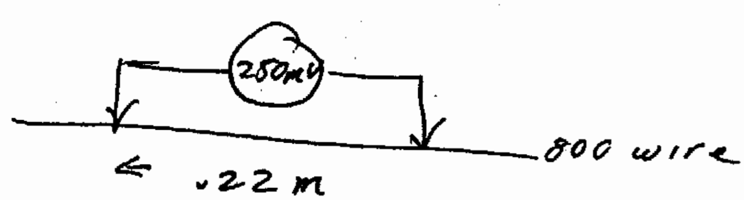



where $n_1 = 10$, $n_2 = 38$

$r = 3.36$ cm $\pm$ .15 cm

$d = 7$ mm $\pm$ 1.5 mm

When I placed 2 cm x 2 cm pieces of Al foil directly above the center of the coil, the $F_{grav} = n \rho g A t$, $n = \#$ of folds of the foil acted to counteract the repulsive force of the coils. I turned up the voltage and measured the current across the 800 wire




$$i = \frac{V_{measured}}{R_{wire}}$$

where $R_{wire} = (1.02 \frac{\Omega}{m})(.22\text{ m}) = .2244\ \Omega$

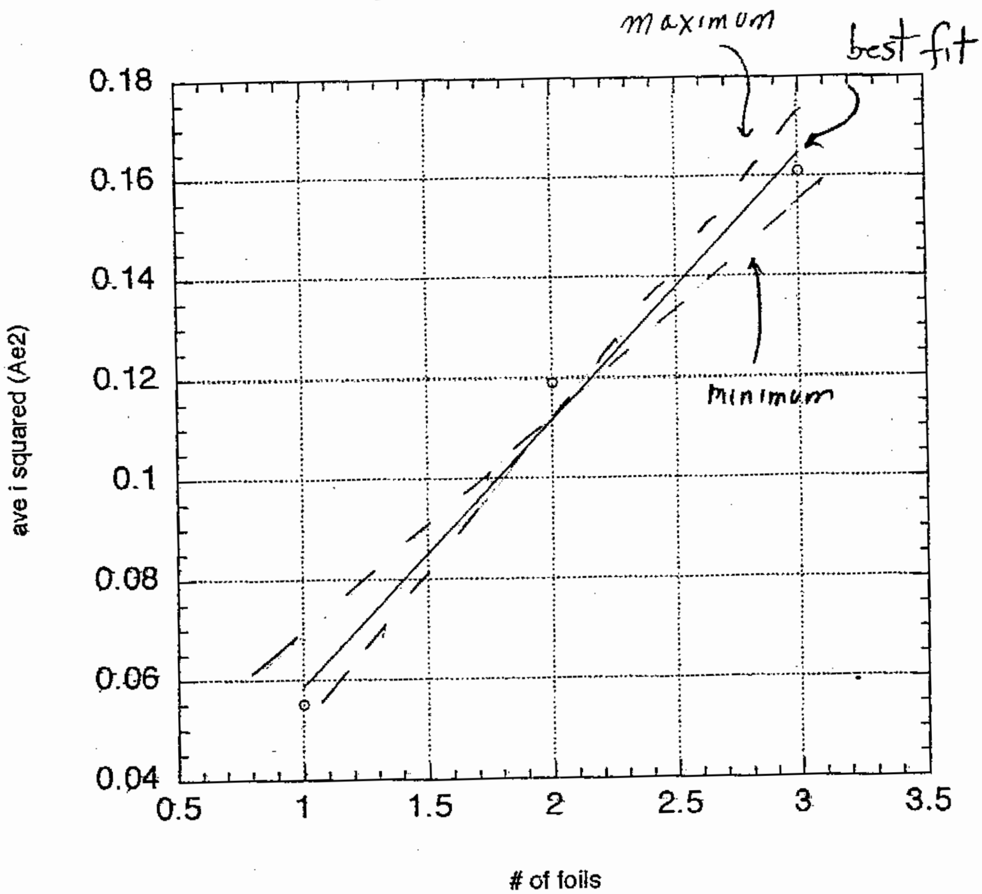
My data and plot of $i^2$ vs $n$ is shown.

Expt MF data and graph:
4/7/00

| | # of foils | ave voltage (mV) | ave current (A) | ave i squared (Ae2) |
|---|---|---|---|---|
| 0 | 1 | 52.700 | 0.235 | 0.0551 |
| 1 | 2 | 77.300 | 0.344 | 0.119 |
| 2 | 3 | 90.000 | 0.401 | 0.161 |

| | # of foils | trial 1 (mV) | trial 2 (mV) | trial 3 (mV) | ave (mV) |
|---|---|---|---|---|---|
| 0 | 1 | 52.0 | 51.0 | 55.0 | 52.7 |
| 1 | 2 | 78.0 | 80.0 | 74.0 | 77.3 |
| 2 | 3 | 90.0 | 88.0 | 92.0 | 90.0 |

$i^2 = 0.0058 + 0.0529\,n$ R= 0.993



When the forces balanced,

$$\frac{\mu_0 n_1 n_2 i^2 r}{d} = n \rho g A t$$

$$i^2 = \frac{\rho g A t d}{\mu_0 n_1 n_2 r} n$$

$$\alpha \equiv \text{slope} = \frac{\rho g A t d}{\mu_0 n_1 n_2 r}$$

From by graph, the $\alpha \equiv$ slope $= .0529 \frac{A^2}{\#}$

I drew maximum and minimum lines which fit the data and found

$(\alpha_{max}) = .060 A^2/m^2$ $\quad (\alpha_{min}) = .043 A^2/m^2$

So I estimate the error of my slope to be

$$\frac{\Delta\alpha}{\alpha} \simeq .16$$

where I averaged $\frac{(\alpha_{max} - \alpha) + (\alpha - \alpha_{min})}{2} = \Delta\alpha = .0087 \frac{A^2}{\#^2}$

Since $\mu_0 = \frac{1}{(\text{slope})} \frac{\rho g A t d}{n_1 n_2 r}$ where

$\rho = 2.7 \times 10^3 \, kg/m^3$, $A = 4 \times 10^{-4} m^2$, $t = 1.8 \times 10^{-5} m$

$d = 7 \times 10^{-3} m$, $n_1 = 10$, $n_2 = 38$, $r = 3.36 \times 10^{-2} m$

$$\mu_0 = \frac{1}{(.0529 \frac{A^2}{\#})} \frac{(2.7 \times 10^3 \frac{kg}{m^3})(9.8 \frac{m}{s^2})(4 \times 10^{-4} m^2)(1.8 \times 10^{-5} m)(7 \times 10^{-3} m)}{(10)(38)(3.36 \times 10^{-2} m)}$$

$$= 1.97 \times 10^{-6} \frac{\text{tesla-m}}{A}$$

The accepted value for $\mu_o$ is

$$(\mu_o)_{th} = 4\pi \times 10^{-7} \frac{T\text{-}m}{A} = 1.26 \times 10^{-6}.$$

Now the biggest error is in the distance between the coils

$$\frac{\Delta d}{d} = \frac{1.5 \text{ mm}}{7 \text{ m}} = .21$$

The approximation to the B field

$$\frac{\Delta B}{B} \simeq .05 \qquad \text{(see A.P. French: note on MF)}$$

So

$$\frac{\Delta \mu_o}{\mu_o} = \left(\left(\frac{\Delta \alpha}{\alpha}\right)^2 + \left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta B}{B}\right)^2\right)^{1/2}$$

$$= \left((.16)^2 + (.21)^2 + (.05)^2\right)^{1/2} = .26$$

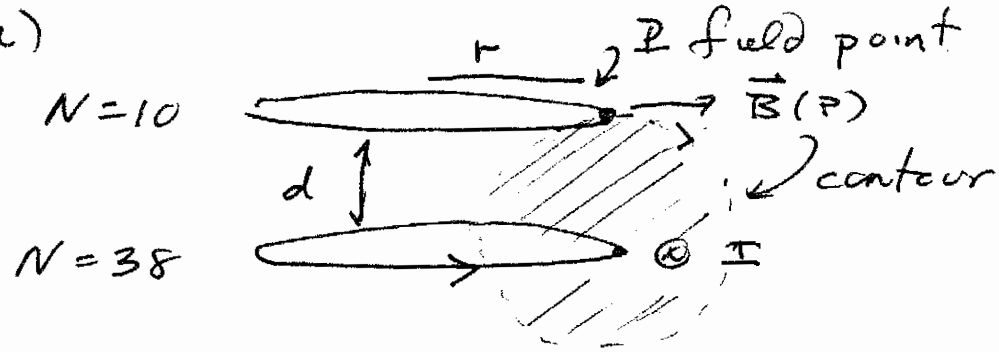
$$\frac{(\mu_o)_{expt} - (\mu_o)_{th}}{(\mu_o)_{th}} = \frac{1.97 \times 10^{-6} - 1.26 \times 10^{-6}}{1.26 \times 10^{-6}}$$

$$= .56.$$

The discrepancy between the experimental result and the theoretical results suggest that there is a systematic error. The most likely place is the thickness of the foil $t = 1.8 \times 10^{-5}$ m.

## Problem 2

a)




From Ampere's Law

| $\oint \vec{B} \cdot d\vec{r}$ | $= \mu_0 I_{enc}$ |
|---|---|
| $B\, 2\pi d$ | $= \mu_0 (38)(I)$ |

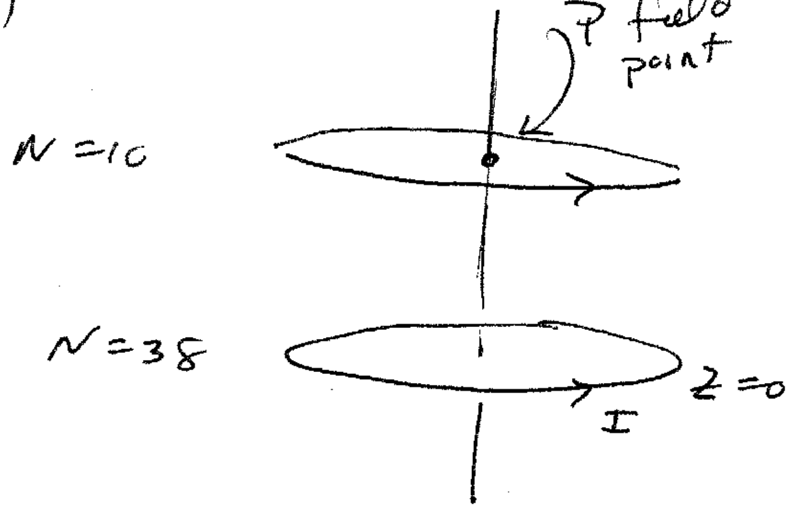
$$B = \frac{38 \mu_0 I}{2\pi d} = \frac{(38)(2\times10^{-7}\frac{T\text{-}m}{A})(.5A)}{(5\times10^{-3}m)}$$

where $\frac{\mu_0}{2\pi} = 2\times10^{-7}\,\frac{T\text{-}m}{A}$

$$B = 7.6\times10^{-4}\,T = 7.6 \text{ gauss}$$

In this calculation, we are approximating the magnetic field of the N=38 turn coil as the field of a long straight wire for field points with $d \ll r$

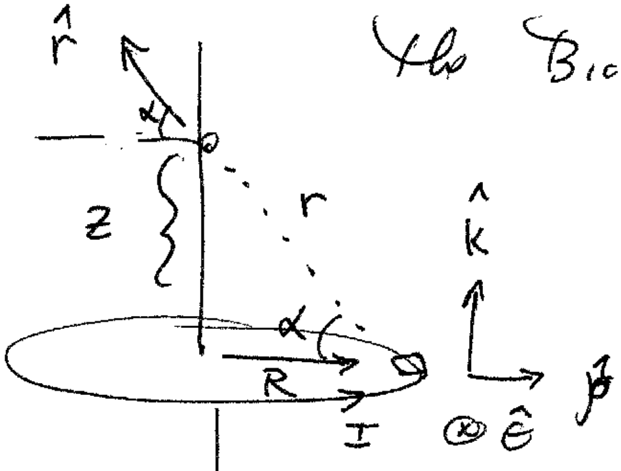
b)




When the coils are in series the current flows in the same direction

$$\vec{B}(P) = \vec{B}_{38}(z) + \vec{B}_{10}(\text{center}) \quad \text{superposition}$$

So let's calculate the $\vec{B}_{38}(z)$ using the Biot-Savart Law




$$\vec{B} = \int_{\text{coil}} \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

$$I\, d\vec{l} = N_{38} I R\, d\theta\, \hat{\theta}$$

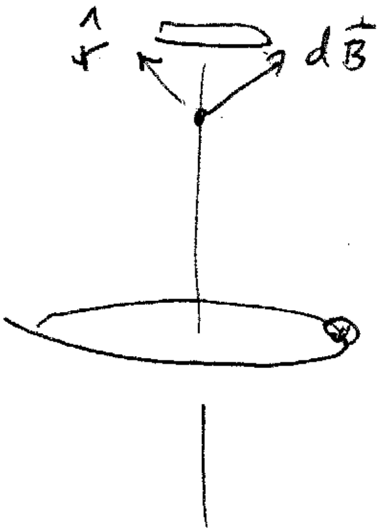
$$\hat{\rho} \times \hat{\theta} = \hat{k}$$
$$\hat{\theta} \times \hat{k} = \hat{\rho}$$

$$\hat{r} = \cos\alpha(-\hat{\rho}) + \sin\alpha\, \hat{k}$$

$$r = (R^2 + z^2)^{1/2}$$

$$\vec{B} = \int_{\theta=0}^{\theta=2\pi} \frac{\mu_0}{4\pi} \frac{N_{38} I R\, d\theta\, \hat{\theta} \times (-\cos\alpha\, \hat{\rho} + \sin\alpha\, \hat{k})}{r^2} \qquad \cos\alpha = \frac{R}{r}, \quad \sin\alpha = \frac{z}{r}$$

$$= \frac{\mu_0}{4\pi} N_{38} I R \left( \int_0^{2\pi} \frac{d\theta \cos\alpha\, \hat{k}}{r^2} + \int_0^{2\pi} \frac{d\theta \sin\alpha\, \hat{\rho}}{r^2} \right)$$



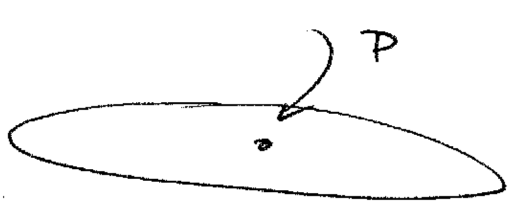

As we go integrate around the circle, $d\vec{B}$ sweeps out a cone the tangential component sums to zero, leaving only a vertical non-zero component.

$$\vec{B}(z) = \frac{\mu_0}{4\pi} N_{38} I R \int_0^{2\pi} \frac{d\theta\, R}{r^3}\ \hat{k}$$

$$= \frac{\mu_0}{4\pi} N_{38} \frac{I R^2}{r^3} 2\pi\ \hat{k}$$

$$\vec{B}_{38}(z) = \frac{\mu_0}{4\pi} N_{38} \frac{I R^2}{(R^2+z^2)^{3/2}} 2\pi\ \hat{k}$$

For the 10-turn coil




$z = 0$

$$\vec{B}_{10}(\text{center}) = \frac{\mu_0}{2} N_{10} \frac{I}{R}\ \hat{k}$$

$$\vec{B}^{total} = \vec{B}_{38}(z) + \vec{B}_{10}(\text{center})$$

$$= \left( \frac{\mu_0}{2} N_{38} \frac{I R^2}{(R^2+z^2)^{3/2}} + \frac{\mu_0}{2} \frac{N_{10} I}{R} \right) \hat{k}$$

$$= \left(\frac{\mu_0 I}{2}\right)\left( \frac{N_{38} R^2}{(R^2+z^2)^{3/2}} + \frac{N_{10}}{R} \right) \hat{k} \quad , \quad \frac{\mu_0}{2\pi} = 2\times10^{-7} \frac{T\cdot m}{A}$$

$$= \left(2\times10^{-7} \frac{T\cdot m}{A}\right)(\pi)(0.5A)\left( \frac{(38)(6\times10^{-2}m)^2}{\left((6\times10^{-2}m)^2+(0.5\times10^{-2}m)^2\right)^{3/2}} + \frac{10}{(6\times10^{-2}m)} \right)\hat{k}$$

where $z = 0.5\times10^{-2} m$

$$\vec{B} = 2.5\times10^{-4} T \hat{k}$$

$$= (2.5 \text{ gauss})\,\hat{k}$$