

**Topics:** Magnetic Fields: Creating Magnetic Fields – Biot-Savart

**Related Reading:**

Course Notes (Liao et al.): Sections 9.1 – 9.2  
 Serway and Jewett: Sections 30.1 – 30.2  
 Giancoli: Sections 28.1 – 28.3

**Experiments:** (6) Magnetic Force on Current-Carrying Wires

## Topic Introduction

Last class we focused on the forces that moving charges *feel* when in a magnetic field. Today we will extend this to currents in wires, and then discuss how moving charges and currents can also *create* magnetic fields. The presentation is analogous to our discussion of charges creating electric fields. We first describe the magnetic field generated by a single charge and then proceed to collections of moving charges (currents), the fields from which we will calculate using superposition – just like for continuous charge distributions.

### Lorenz Force on Currents

Since a current is nothing more than moving charges, a current carrying wire will also feel a force when placed in a magnetic field:  $\vec{F} = I\vec{L} \times \vec{B}$  (where  $I$  is the current, and  $\vec{L}$  is a vector pointing along the axis of the wire, with magnitude equal to the length of the wire).

### Field from a Single Moving Charge

Just as a single electric charge creates an electric field which is proportional to charge  $q$  and falls off as  $r^{-2}$ , a single moving electric charge additionally creates a magnetic field given by

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{r^2}$$

Note the similarity to Coulomb's law for the electric field – the field is proportional to the charge  $q$ , obeys an inverse square law in  $r$ , and depends on a constant, the permeability of free space  $\mu_0 = 4\pi \times 10^{-7}$  T m/A. The difference is that the field no longer points along  $\hat{r}$  but is instead perpendicular to it (because of the cross product).

### Field from a Current: Biot-Savart Law

We can immediately switch over from discrete charges to currents by replacing  $q \vec{v}$  with  $I d\vec{s}$ :

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$

This is the Biot-Savart formula, and, like the differential form of Coulomb's Law, provides a generic method for calculating fields – here magnetic fields generated by currents. The  $ds$  in this formula is a small length of the wire carrying the current  $I$ , so that  $I ds$  plays the same role that  $dq$  did when we calculated electric fields from continuous charge distributions. To find the total magnetic field at some point in space you integrate over the current distribution (e.g. along the length of the wire), adding up the field generated by each little part of it  $ds$ .

### Right Hand Rules

Because of the cross product in the Biot-Savart Law, the direction of the resulting magnetic field is not as simple as when we were working with electric fields. In order to quickly see what direction the field will be in, or what direction the force on a moving particle will be in,

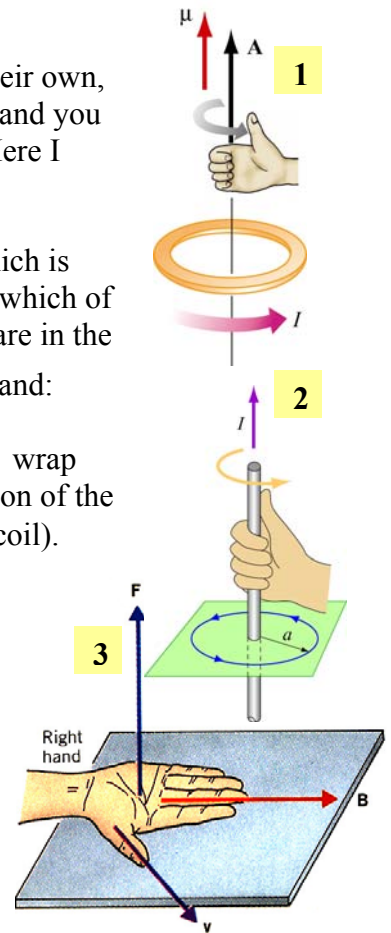
we can use a “Right Hand Rule.” At times it seems that everyone has their own, unique, right hand rule. Certainly there are a number of them out there, and you should feel free to use whichever allow you to get the correct answer. Here I describe the three that I use (starting with one useful for today’s lab).

The important thing to remember is that cross-products yield a result which is perpendicular to both of the input vectors. The only open question is in which of the two perpendicular directions will the result point (e.g. if the vectors are in the floor does their cross product point up or down?). Using your RIGHT hand:

1) For determining the direction of the dipole moment of a coil of wire: wrap your fingers in the direction of current. Your thumb points in the direction of the North pole of the dipole (in the direction of the dipole moment  $\mu$  of the coil).

2) For determining the direction of the magnetic field generated by a current: fields wrap around currents the same direction that your fingers wrap around your thumb. At any point the field points tangent to the circle your fingers will make as you twist your hand keeping your thumb along the current.

3) For determining the direction of the force of a field on a moving charge: open your hand perfectly flat. Put your thumb along  $v$  and your fingers along  $B$ . Your palm points along the direction of the force.



## Important Equations

Force on Current-Carrying Wire of Length  $L$ :

$$\vec{F} = I\vec{L} \times \vec{B}$$

Biot-Savart – Field created by moving charge; current:  $\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{r^2}$ ;  $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$

### Experiment 6: Magnetic Force on Current-Carrying Wires

**Preparation:** Read lab write-up.

In this lab you will be able to feel the force between a current carrying wire and a permanent magnet. Before making the measurements try to determine what kind of force you should feel. For straight wires the easiest way to determine this is to use the formula  $\vec{F} = I\vec{L} \times \vec{B}$  and to determine what direction the field is in remembering that the permanent magnet is a dipole, creating fields which loop from its North to its South pole. For a coil of wire, the easiest way to determine the force is to think of the coil as a magnet itself. A coil of wire creates a field very much like that you measured last time for the Earth and the bar magnets. In fact, we will treat a coil of wire just like a dipole. So to determine the force on the coil, replace it in your mind with a bar magnet (oriented with the N pole pointing the way your thumb does when you wrap your fingers in the direction of current) and ask “How will these two magnets interact?”