Class 13: Outline

Hour 1:

Concept Review / Overview

PRS Questions – possible exam questions

Hour 2:

Sample Exam

EXAM Thursday: 7:30 – 9 pm

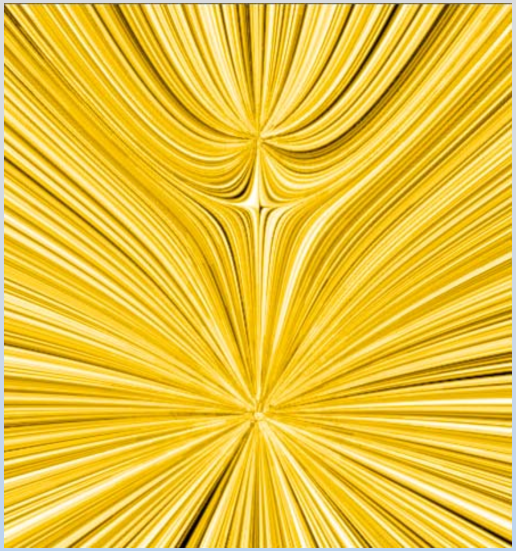
Exam 1 Topics

- Fields (visualizations)
- Electric Field & Potential
  - Discrete Point Charges
  - Continuous Charge Distributions
  - Symmetric Distributions – Gauss's Law
- Conductors
- Capacitance
  - Calculate for various geometries
  - Effects of dielectrics
  - Energy storage

General Exam Suggestions

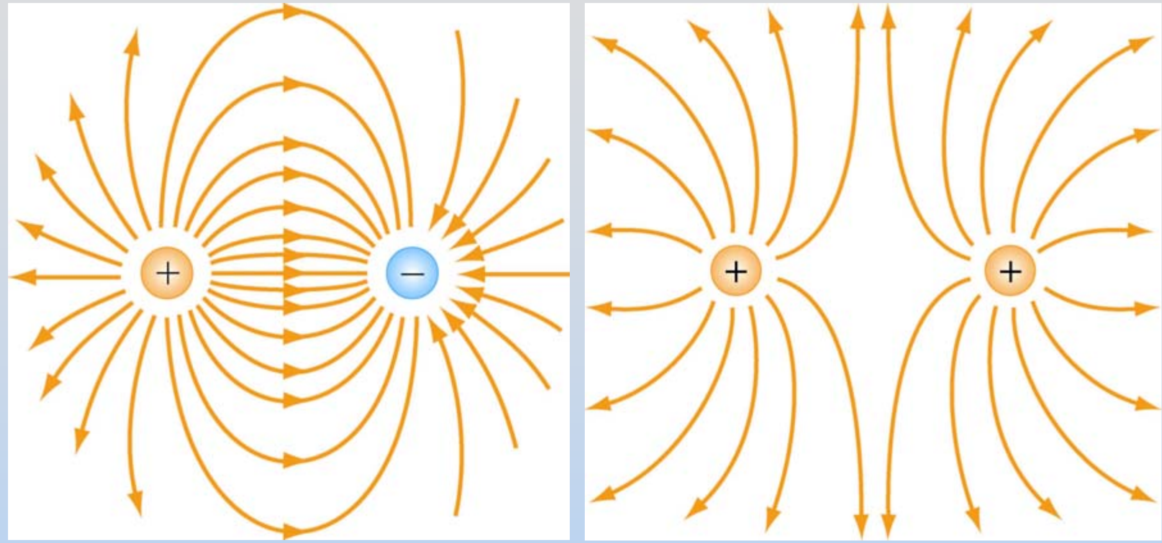
- You should be able to complete every problem
  - If you are confused, ask
  - If it seems too hard, think some more
  - Look for hints in other problems
  - If you are doing math, you’re doing too much
- Read directions completely (before & after)
- Write down what you know before starting
- Draw pictures, define (label) variables
  - Make sure that unknowns drop out of solution
- Don’t forget units!

Fields



Grass Seeds

Know how to read



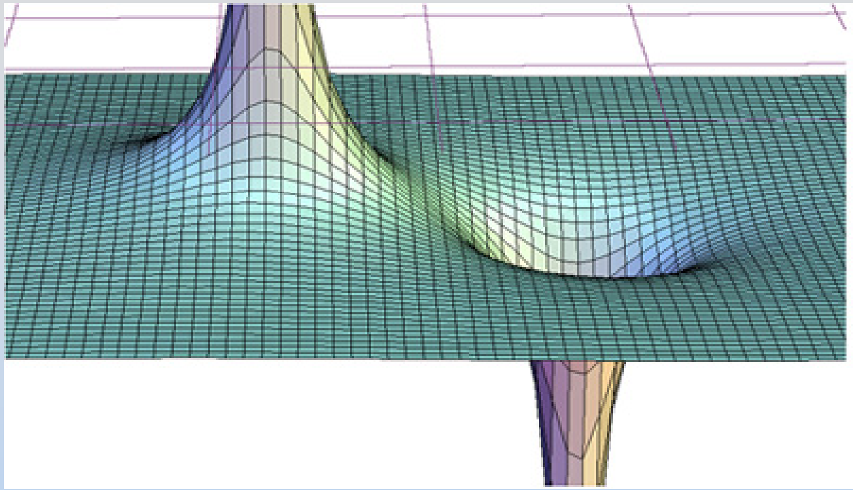
Field Lines

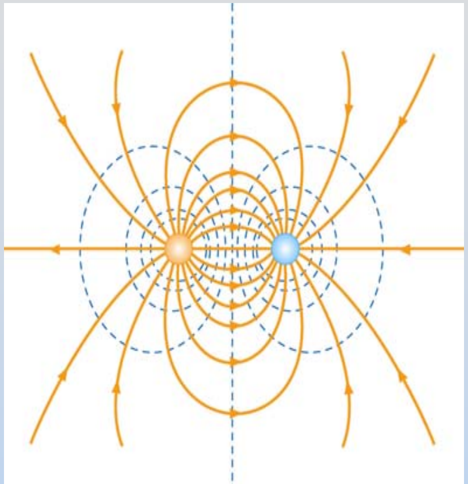
Know how to draw

- Field line density tells you field strength
- Lines have tension (want to be straight)
- Lines are repulsive (want to be far from other lines)
- Lines begin and end on sources (charges) or ∞

PRS Questions: Fields

E Field and Potential: Creating





A point charge *q* creates a field and potential around it:

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}};\;\; V = k_e \frac{q}{r}$$

Use superposition for systems of charges

They are related:

$$\vec{\mathbf{E}} = -\nabla V;\;\; \Delta V \equiv V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$$

E Field and Potential: Creating

<u>Discrete set of point charges:</u>

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}; \;\; V = k_e \frac{q}{r}$$

Add up from each point charge

<u>Continuous charge distribution</u>:

$$d\vec{\mathbf{E}} = k_e \frac{dq}{r^2}\hat{\mathbf{r}}; \;\; dV = k_e \frac{dq}{r}$$

Break charged object into small pieces, *dq*, and integrate

Continuous Sources: Charge Density

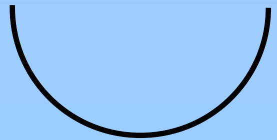
Charge Densities:

$$\lambda = \frac{Q}{L} \qquad \sigma = \frac{Q}{A} \qquad \rho = \frac{Q}{V}$$

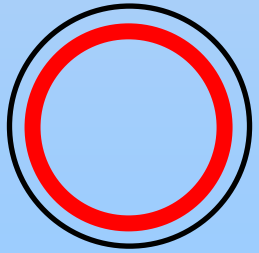
$$dQ = \lambda\, dL \qquad dQ = \sigma\, dA \qquad dQ = \rho\, dV$$

Don’t forget your geometry:

$$dL = dx$$



$$dL = R d\theta$$



$$dA = 2\pi r dr$$

$$dV_{cyl} = 2\pi r l dr$$

$$dV_{sphere} = 4\pi r^2 dr$$

E Field and Potential: Creating

Discrete set of point charges:

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}; \;\; V = k_e \frac{q}{r}$$

Add up from each point charge

Continuous charge distribution:

$$d\vec{\mathbf{E}} = k_e \frac{dq}{r^2}\hat{\mathbf{r}}; \;\; dV = k_e \frac{dq}{r}$$

Break charged object into small pieces, *dq*, and integrate

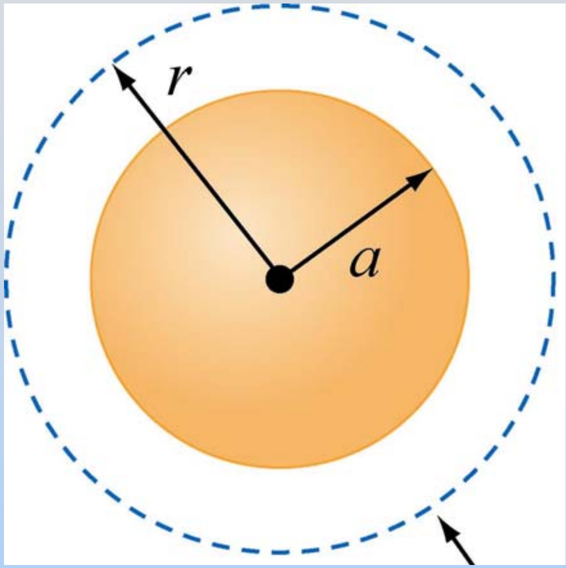
Symmetric charged object:

$$\oint\!\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}; \; \Delta V \equiv -\int \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$$

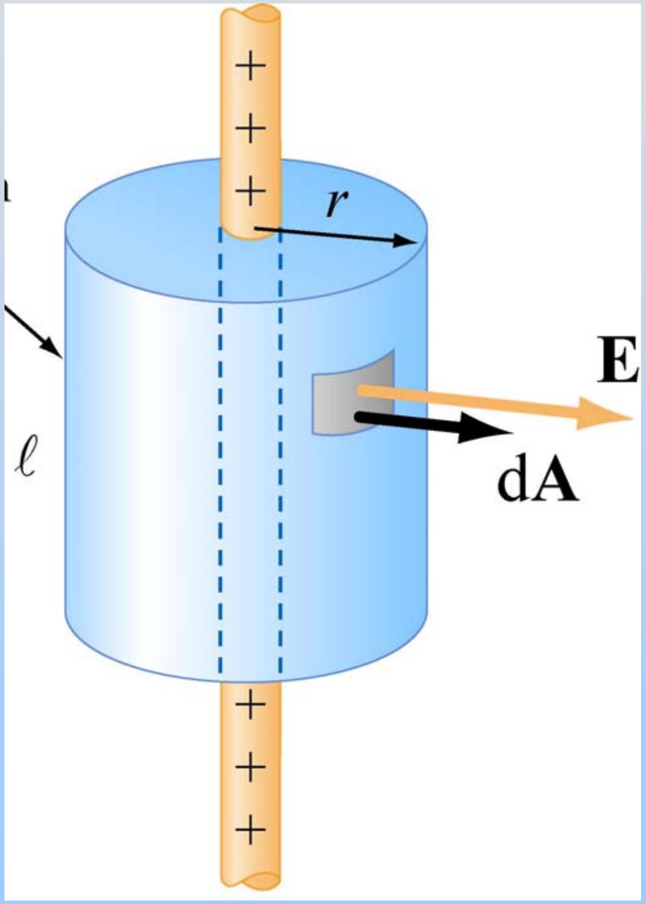
Use Gauss' law to get E everywhere, then integrate to get V

Gauss's Law:

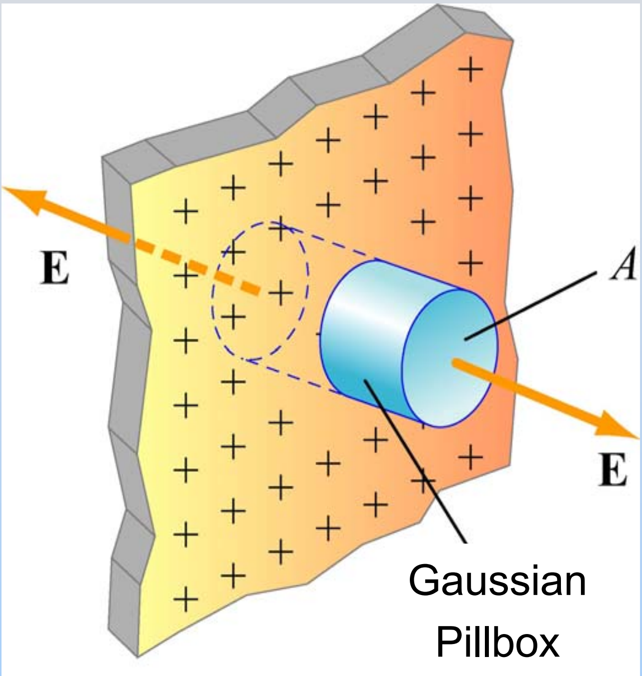
$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$$



Spherical Symmetry



Cylindrical Symmetry



Planar Symmetry

E Field and Potential: Effects

If you put a charged particle, *q*, in a field:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

To move a charged particle, *q*, in a field:

$$W = \Delta U = q\Delta V$$

PRS Questions: Electric Fields and Potential

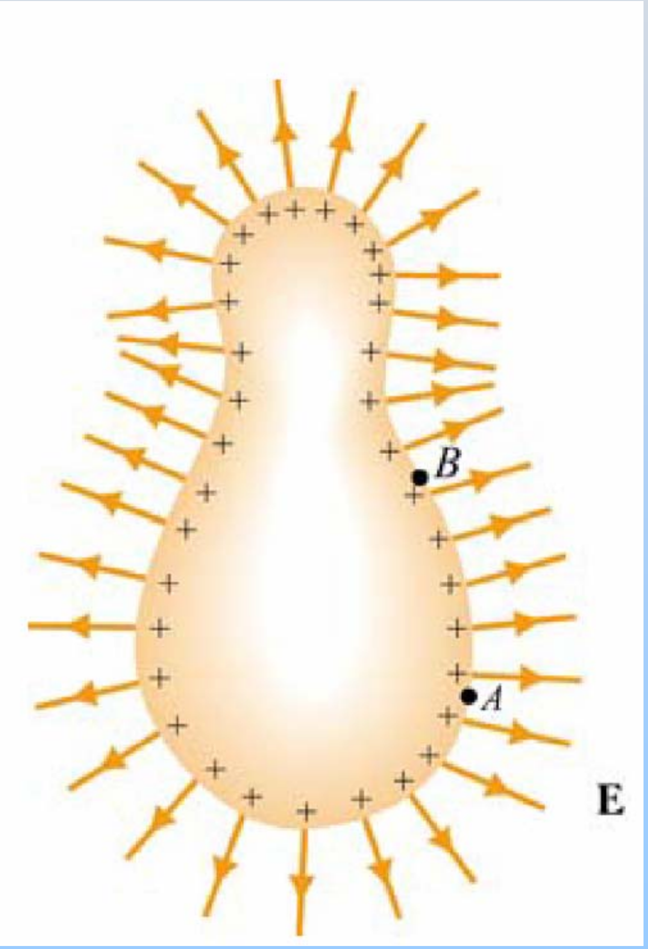
Conductors in Equilibrium

Conductors are equipotential objects:

1) E = 0 inside
2) Net charge inside is 0
3) E perpendicular to surface
4) Excess charge on surface

$$E = \sigma / \varepsilon_0$$

5) Shielding – inside doesn't "talk" to outside



PRS Questions: Conductors

Capacitors

Capacitance

$$C = \frac{Q}{|\Delta V|}$$

To calculate:

1) Put on arbitrary ±Q
2) Calculate E
3) Calculate ΔV

In Series & Parallel

$$\frac{1}{C_{eq,\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq,\text{parallel}} = C_1 + C_2$$

Energy

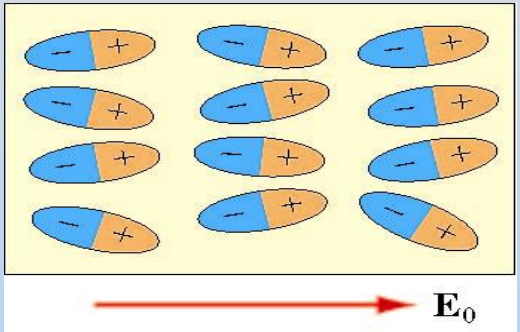
$$U = \frac{Q^2}{2C} = \frac{1}{2}Q|\Delta V| = \frac{1}{2}C|\Delta V|^2 = \iiint u_E \, d^3r = \iiint \frac{\varepsilon_o E^2}{2} d^3r$$

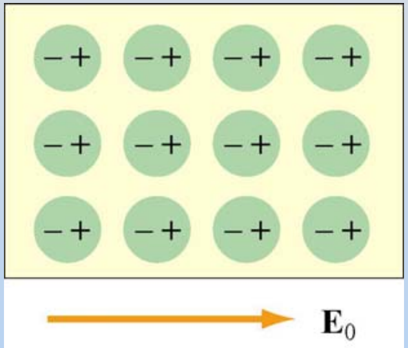
PRS Questions: Capacitors

Dielectrics

Dielectrics locally *<u>weaken</u>* the electric field

$$E = \frac{E_0}{\kappa}; \ \kappa \geq 1$$





Inserted into a capacitor: $C = \kappa C_0$

$$C = \frac{Q}{|\Delta V|}$$

Hooked to a battery? Q increases

Not hooked up? V decreases

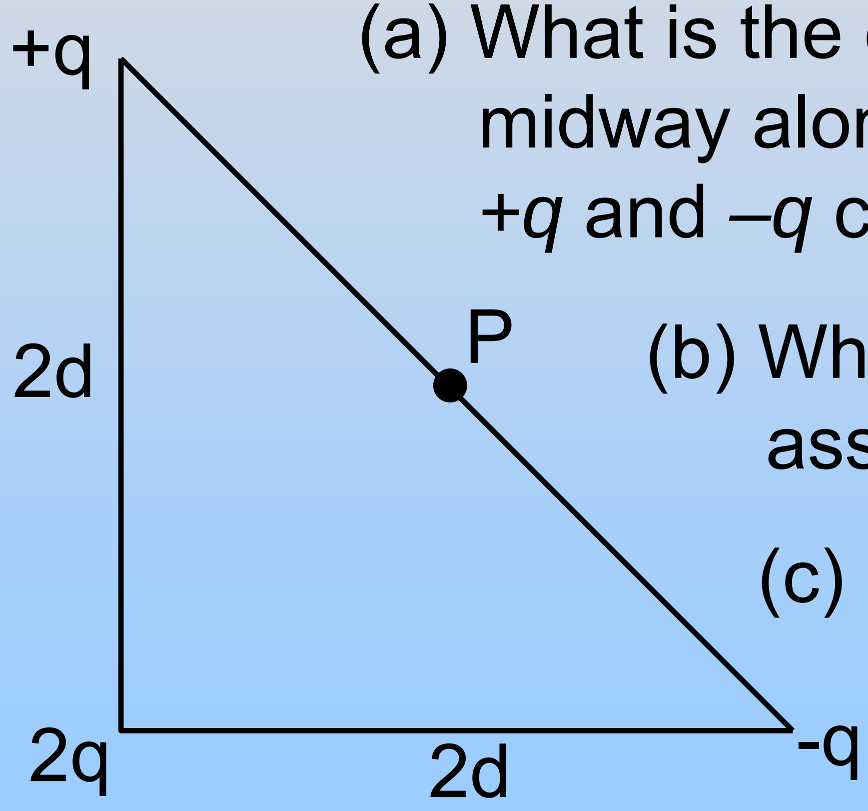
PRS Questions: Dielectrics

SAMPLE EXAM:

The real exam has 5 concept, 3 analytical questions

Q: Point Charges

A right isosceles triangle of side 2*d* has charges *q*, +2*q* and -*q* arranged on its vertices (see sketch).



(a) What is the electric field at point *P*, midway along the line connecting the +*q* and –*q* charges?

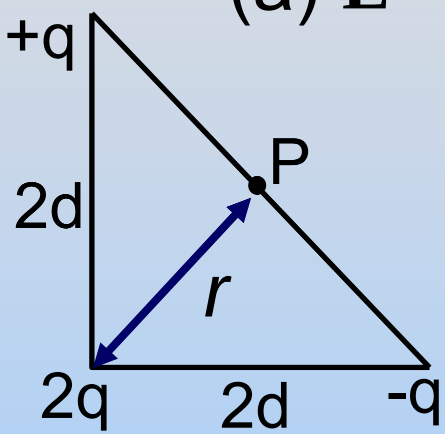
(b) What is the potential at *P*, assuming V(∞)=0?

(c) How much work to bring a charge -5Q from ∞ to P?

A: Point Charges

All charges a distance $r = \sqrt{2}d$ from P

(a) $\vec{\mathbf{E}} = \sum \frac{kQ}{r^3}\vec{\mathbf{r}} \;\rightarrow\; E_x = \frac{k}{r^3}\sum Qx; \quad E_y = \frac{k}{r^3}\sum Qy$



$$E_x = \frac{k}{r^3}\big(qd + 2qd + (-q)(-d)\big) = \frac{4kqd}{r^3}$$

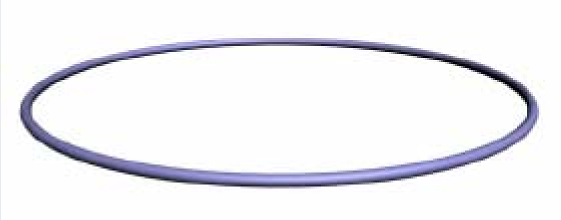
$$E_y = \frac{k}{r^3}\big(q(-d) + 2qd + (-q)d\big) = 0$$

(b) $V = \sum \frac{kQ}{r} = \frac{k}{r}\left(q + 2q - q\right) = \boxed{\frac{2kq}{r} = V}$

(c) $W = \Delta U = \left(-5Q\right)\Delta V = \left(-5Q\right)V\left(P\right) = \boxed{\frac{-10kqQ}{r} = W}$

Q: Ring of Charge

A thin rod with a uniform charge per unit length λ is bent into the shape of a circle of radius *R*



a) Choose a coordinate system for the rod. Clearly indicate your choice of origin, and axes on the diagram above.

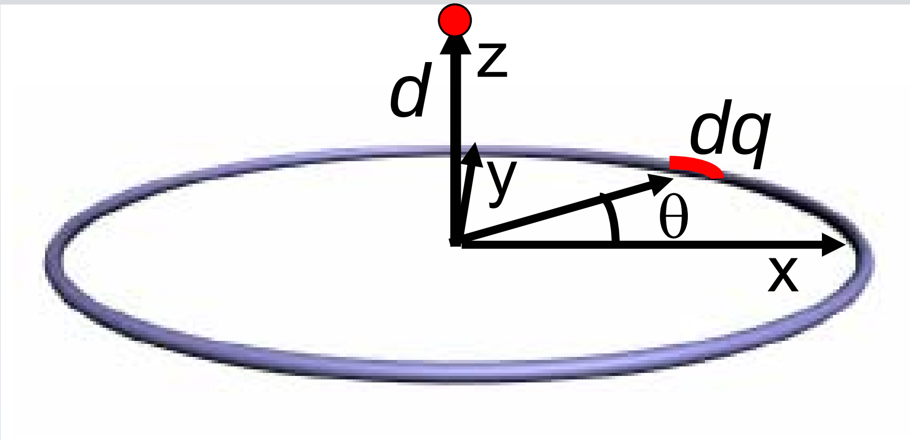
b) Choose an infinitesimal charge element *dq* . Find an expression relating *dq* , λ, and your choice of length for *dq* .

c) Find the vector components for the contribution of *dq* to the electric field along an axis perpendicular to the plane of the circle, a distance *d* above the plane of the circle. The axis passes through the center of the circle. Express the vector components in terms of your choice of unit vectors

d) What is the direction and magnitude of the electric field along the axis that passes through the center of the circle, perpendicular to the plane of the circle, and a distance *d* above the plane of the circle.

e) What is the potential at that point, assuming $V(\infty)=0$?

A: Ring of Charge



a) Origin & axes as pictured

b) $dq = \lambda d\ell = \lambda R d\theta$

c) $d\vec{\mathbf{E}} = \frac{k\,dq}{r^3}\vec{\mathbf{r}}$

$$\vec{\mathbf{r}} = -R\cos(\theta)\hat{\mathbf{i}} - R\sin(\theta)\hat{\mathbf{j}} + d\hat{\mathbf{k}}; \quad r = \sqrt{R^2 + d^2}$$

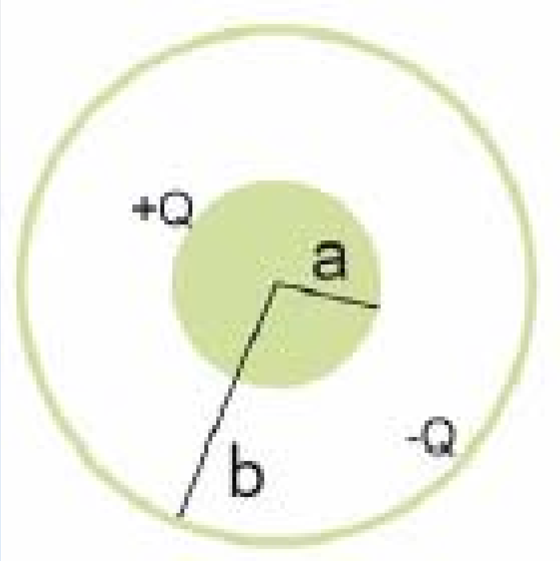
d) Horizontal components cancel, only find E_z

$$E_z = \int dE_z = \int \frac{k\,dq}{r^3} d = \frac{kd}{r^3}\int_{\theta=0}^{2\pi} \lambda R\,d\theta = \boxed{\frac{kd\lambda R}{r^3}2\pi}$$

e) Find the potential by same method:

$$V(d) = \int dV = \int \frac{k\,dq}{r} = \frac{k}{r}\int_{\theta=0}^{2\pi} \lambda R\,d\theta = \boxed{\frac{k\lambda R}{r}2\pi}$$

Q: Spherical Capacitor



A conducting solid sphere of radius *a*, carrying a charge +*Q* is surrounded by a thin conducting spherical shell (inner radius *b*) with charge -*Q* .

a) What is the direction and magnitude of the electric field **E** in the three regions below. Show how you obtain your expressions.

1. $r < a$ 2. $a < r < b$ 3. $r > b$

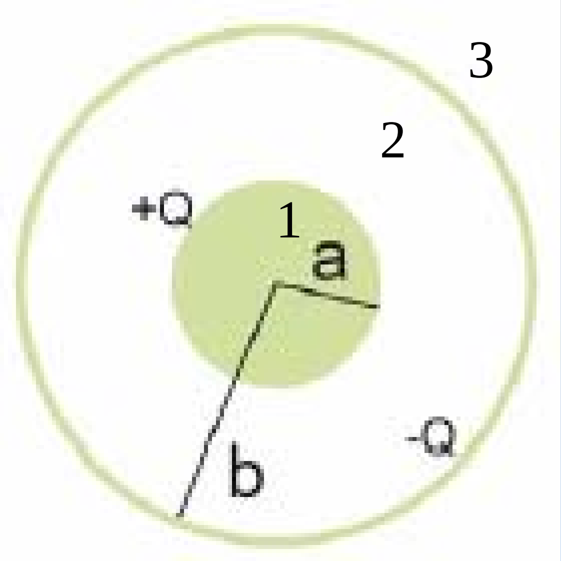
b) What is the electric potential V(r) in these same three regions. Take the electric potential to be zero at ∞.

c) What is the electric potential difference between the outer shell and the inner cylinder, $\Delta V = V(b) - V(a)$?

d) What is the capacitance of this spherical capacitor?

e) If a positive charge +2*Q* is placed anywhere on the inner sphere of radius *a*, what charge appears *on the outside surface* of the thin spherical shell of inner radius *b?*

A: Spherical Capacitor



a) By symmetry **E** is purely radial.
Choose spherical Gaussian surface

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0} = EA = E \cdot 4\pi r^2$$

1&3) $q_{in} = 0 \rightarrow \vec{\mathbf{E}} = 0$ 2) $\vec{\mathbf{E}} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$

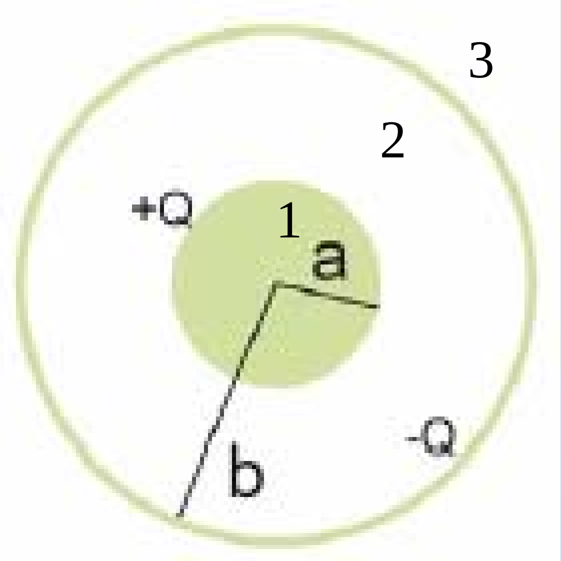
b) For V, always start from where you know it (here, ∞)

3) **E**=0 → *V* constant = 0

2) $V(r) = -\int_b^r \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{b}\right)$

1) **E**=0 → *V* constant = *V*(*a*) → $V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$

A: Spherical Capacitor



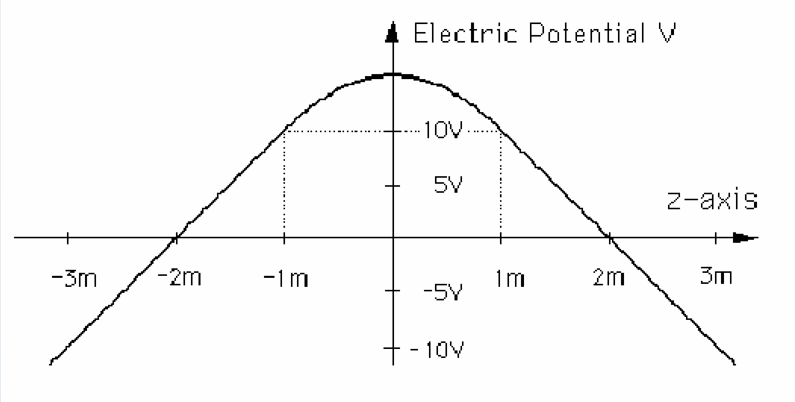
c) $$\Delta V = V(b) - V(a) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

d) $$C = \frac{Q}{|\Delta V|} = \frac{4\pi\varepsilon_0}{\left(a^{-1} - b^{-1}\right)}$$

e) If you place an additional +2Q charge on the inner sphere then you will induce an additional -2Q on the inner surface of the outer shell, and hence a +2Q charge on the outer surface of that shell

Answer: +2Q

Q: Find E from V



The graph shows the variation of an electric potential *V* with distance *z* . The potential *V* does not depend on *x* or *y*. The potential *V* in the region -1 m < *z* < 1 m is given in Volts by the expression **$V(z) = 15 - 5z^2$**. Outside of this region, the electric potential varies linearly with *z*, as indicated in the graph.

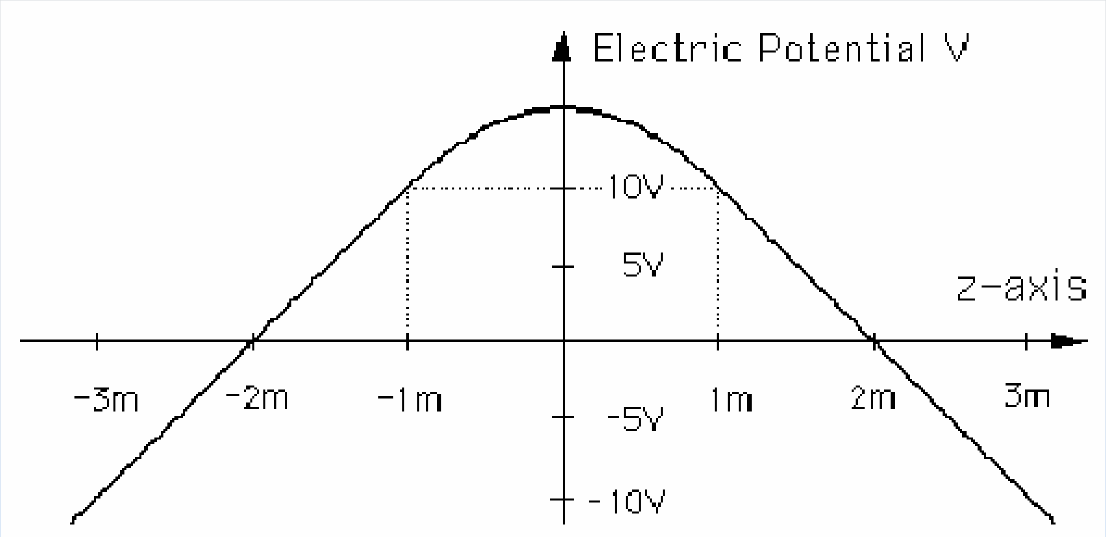
(a) Find an equation for the *z*-component of the electric field, E_z, in the region -1 m < *z* < 1 m.

(b) What is E_z in the region *z* > 1 m? Be careful to indicate the sign

(c) What is E_z in the region *z* < -1 m? Be careful to indicate the sign

(d) This potential is due a slab of charge with constant charge per unit volume ρ_o. Where is this slab of charge located (give the *z*-coordinates that bound the slab)? What is the charge density ρ_o of the slab in C/m^3? Be sure to give clearly both the sign and magnitude of ρ_o.

A: Find E from V



(a) $V(z) = 15 - 5z^2$

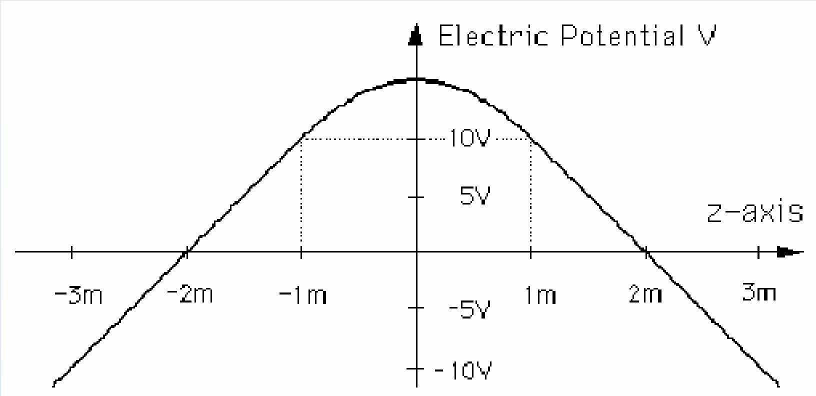
$$E_z = -\frac{\partial V}{\partial z} = 10z$$

(b) (z > 1 m) $$E_z = -\frac{\partial V}{\partial z} = 10\,\mathrm{V/m}$$

(c) (z < -1 m) $$E_z = -\frac{\partial V}{\partial z} = -10\,\mathrm{V/m}$$

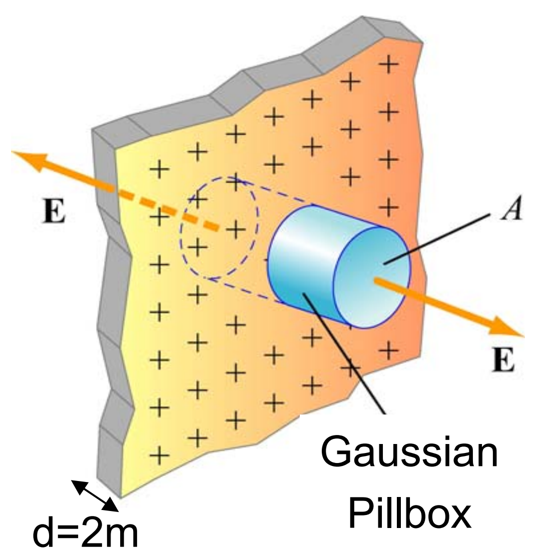
These make sense – the electric field points down the hill

A: Find E from V



(d) Field constant outside slab, so slab from -1m to 1m

The slab is positively charged since E points away

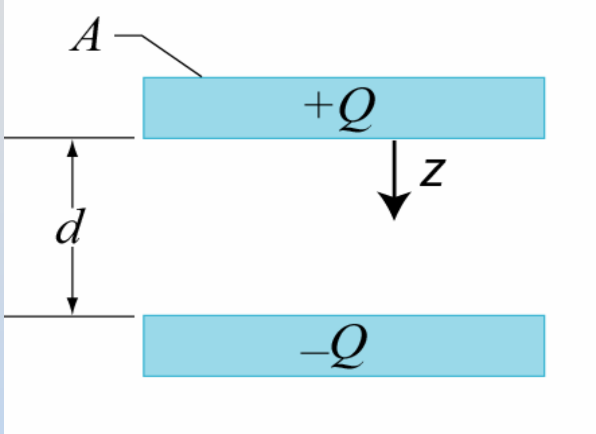


$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0} = E_{Rt}A + E_{Lt}A = 2EA$$

$$2EA = \frac{q_{in}}{\varepsilon_0} = \frac{\rho_0 \text{Volume}_{\text{in}}}{\varepsilon_0} = \frac{\rho_0 A d}{\varepsilon_0}$$

$$\boxed{\rho_0 = \frac{2EA\varepsilon_0}{Ad} = \frac{2(10\,\text{V/m})\varepsilon_0}{(2\text{m})} = 10\varepsilon_0\left[\frac{\text{C}}{\text{m}^3}\right]}$$

Q: Parallel Plate Capacitor



A parallel plate capacitor consists of two conducting plates of area *A*, separated by a distance *d*, with charge +*Q* placed on the upper plate and –Q on the lower plate. The *z*-axis is defined as pictured.

a) What is the direction and magnitude of the electric field **E** in each of the following regions of space: above & below the plates, in the plates and in between the plates.

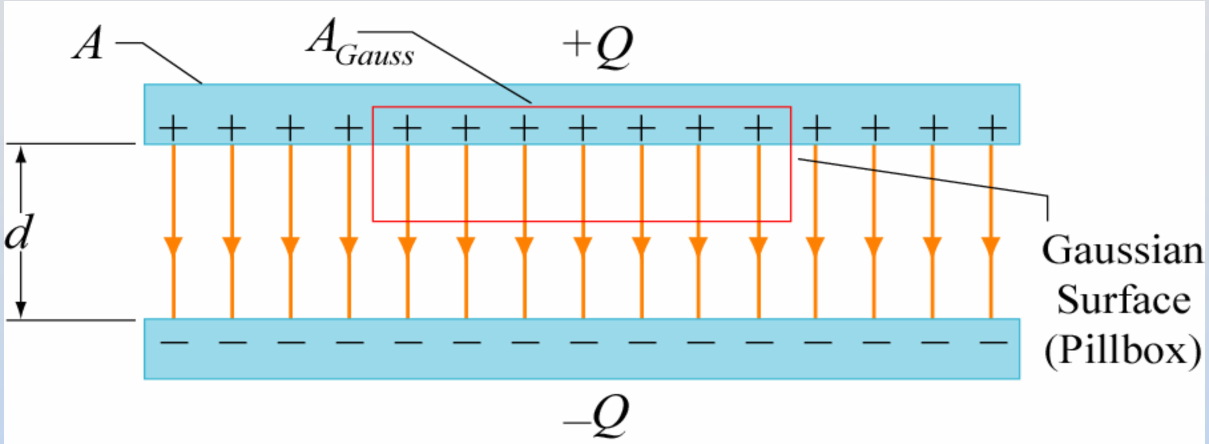
b) What is the electric potential V(*z*) in these same five regions. Take the electric potential to be zero at *z*=0 (the lower surface of the top plate).

c) What is the electric potential difference between the upper and lower plate, $\Delta V = V(0) - V(d)$?

d) What is the capacitance of this capacitor?

e) If this capacitor is now submerged into a vat of liquid dielectric (of dielectric constant κ), what now is the potential V(*z*) everywhere?

A: Parallel Plate Capacitor



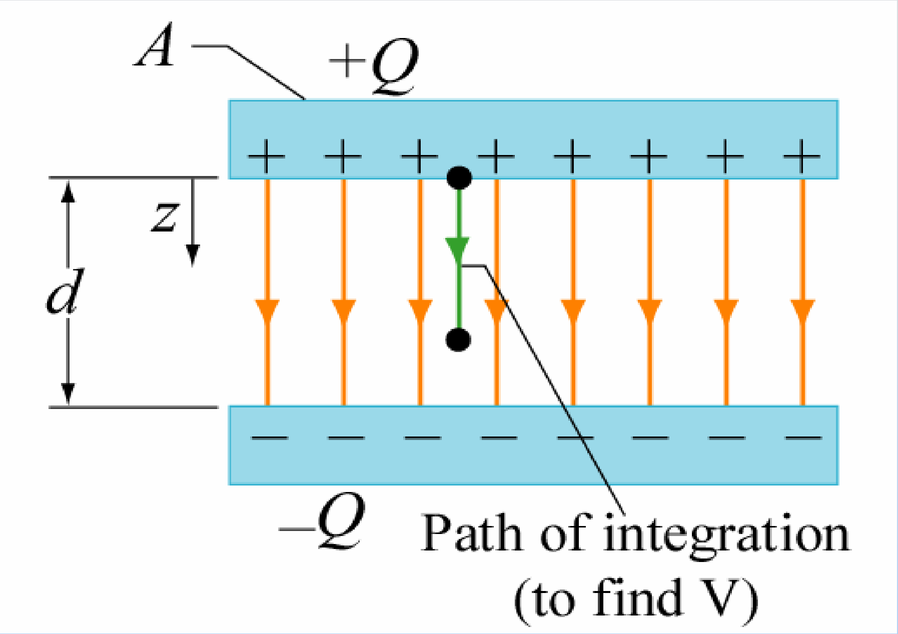
(a) Charges are attracted, so live on inner surface only

Conductors have E=0 inside, and by Gauss's law the only place E≠0 is between the plates:

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0} \qquad E\left(A_{Gauss}\right) = \frac{\sigma A_{Gauss}}{\varepsilon_0} \qquad \boxed{E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \text{ down}}$$

Note that you only need to consider one plate – the other plate was already used (±Q to inner surfaces)

A: Parallel Plate Capacitor



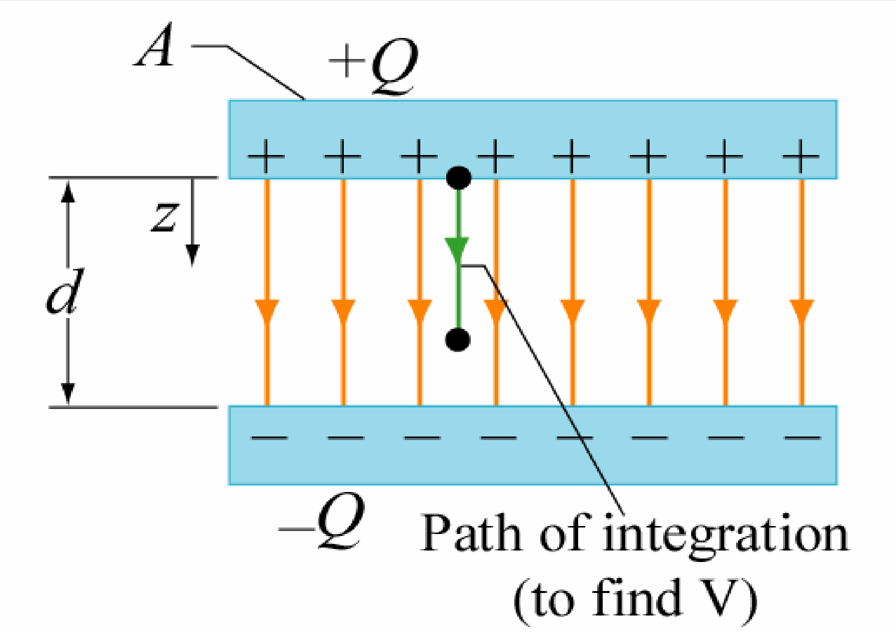
(b) Start where potential is known $V(z = 0) = 0$

Above and inside the top conductor $\mathbf{E} = 0$ so V is constant → $V = 0$

Between plates: $V_{in}(z) = \Delta V = -\int_0^z \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -Ez = -\dfrac{Q}{A\varepsilon_0} z$

In the bottom plate and below (E=0): $V_{below} = -\dfrac{Q}{A\varepsilon_0} d$

A: Parallel Plate Capacitor



(c) $\Delta V = V(0) - V(d) = \dfrac{Qd}{A\varepsilon_0}$

(d) $C = \dfrac{Q}{|\Delta V|} = \dfrac{\varepsilon_0 A}{d}$

(e) The dielectric constant is now everywhere κ.

This reduces the electric field & potential by 1/κ

V above and inside top conductor still 0

$$V_{in}(z) = -\frac{Q}{\kappa A \varepsilon_0} z \qquad V_{below}(z) = -\frac{Q}{\kappa A \varepsilon_0} d$$