Experiment 4: Ohm's Law and *RC* Circuits

OBJECTIVES

1. To learn how to display and interpret signals and circuit outputs using features of *DataStudio*.
2. To investigate Ohm's Law and to determine the resistance of a resistor.
3. To measure the time constants associated with a discharging and charging *RC* (resistive-capacitive, or resistor-capacitor) circuit.

INTRODUCTION

OHM'S LAW

Our main purpose in the Ohm's Law part of the experiment is for you to gain experience with the 750 Interface and the *DataStudio* software, including the signal generator for the 750. We want you to hook up a circuit and a voltage measuring device and look at the voltage across resistors, and get used to what a real circuit looks like. We will have you confirm the relation $V = IR$ in the course of this exercise.

CAPCACITORS

(See the 8.02 *Course Notes*, **Section 5.1**, for a more extensive discussion of capacitors and capacitance.)

In the Capacitor part of this experiment our goals are more complicated because capacitors are more complicated. Capacitors are circuit elements that store electric charge Q, and hence energy, according to the expression

$$Q = C V, \tag{4.1}$$

where V is the voltage across the capacitor and C is the constant of proportionality called the capacitance. The SI unit of capacitance is the farad (after Michael Faraday), 1 farad = (1 coulomb)/(1 volt). Capacitors come in many shapes and sizes, but the basic idea is that a capacitor consists of two conductors separated by a spacing, which may be filled with an insulating material (dielectric). One conductor has charge $+Q$ and the other conductor has charge $-Q$. The conductor with positive charge is at a higher voltage then the conductor with negative charge. Most capacitors have capacitances in the range between picofarads $\left(1\text{pF} = 10^{-12}\text{F}\right)$ and millifarads $\left(1\text{mF} = 10^{-3}\,\text{F} = 1000\,\mu\text{F}\right)$.

Note that we've also used the notation for a *microfarad*, $1\mu F = 10^{-6}\ F = 10^{-3}\ mF$.

CHARGING A CAPACITOR

Consider the circuit shown in Figure 1. The capacitor is connected to a voltage source of constant emf $\mathcal{E}$. At $t = 0$, the switch S is closed. The capacitor initially is uncharged, with $q(t = 0) = 0$. (In the following discussion, we'll represent a time-varying charge as "q" instead of "Q")
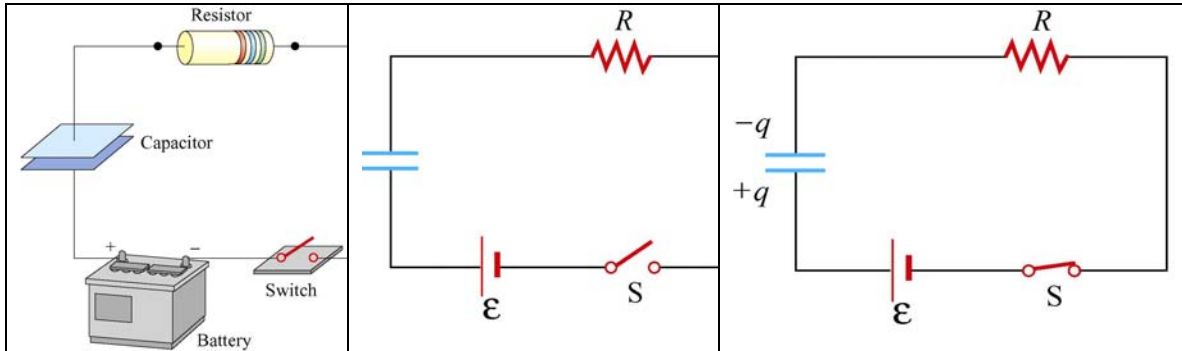

Figure 1 (a) *RC* circuit (b) Circuit diagram for *t* < 0 (c) Circuit diagram for *t* > 0

The expressions for the charge on, and hence voltage across, a charging capacitor, and the current through the resistor, are derived in the 8.02 *Course Notes*, **Section 7.6.1**. This write-up will use the notation $\tau = RC$ for the time constant of either a charging or discharging *RC* circuit.

The capacitor voltage as a function of time is given by $V_C(t) = \dfrac{q(t)}{C} = \mathcal{E}\left(1 - e^{-t/\tau}\right)$;

a graph of this function is given in Figure 2.
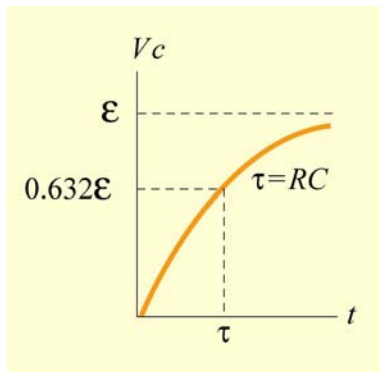
Figure 2 Voltage across capacitor as a function of time for a charging capacitor

The current that flows in the circuit is equal to the derivative with respect to time of the capacitor charge,

$$I = \frac{dq}{dt} = \frac{d}{dt}(CV_C) = \left(\frac{\mathcal{E}}{R}\right)e^{-t/\tau} = I_0\, e^{-t/\tau}\,, \tag{4.2}$$

where I_0 is the initial current that flows in the circuit when the switch was closed at $t = 0$. The graph of current as a function of time is shown in Figure 3:
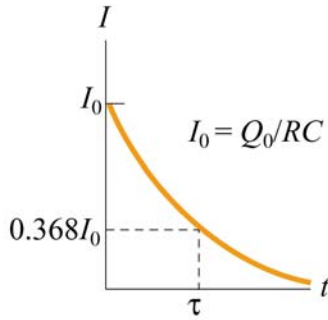
Figure 3 Current as a function of time for a charging capacitor

After one time constant τ has elapsed, the capacitor voltage has increased by a factor of $(1 - e^{-1}) = 0.632$, $V_C(\tau) = \mathcal{E}\left(1 - e^{-1}\right) = 0.632\,\mathcal{E}$ and the current has decreased by a factor of $e^{-1} = 0.368$, $I(\tau) = 0.368\,I_0$.
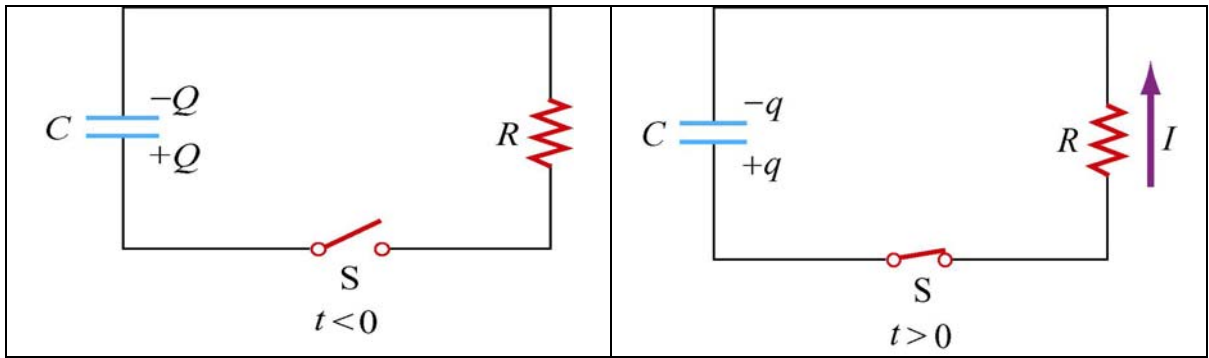
DISCHARGING A CAPACITOR

Suppose we initially charge a capacitor to a charge Q_0 through some charging circuit. At time $t = 0$ the switch is closed (Figure 4). The capacitor will begin to discharge.

The expressions for the charge on, and hence voltage across, a discharging capacitor, and the current through the resistor, are derived in the 8.02 *Course Notes*, **Section 7.6.1**.
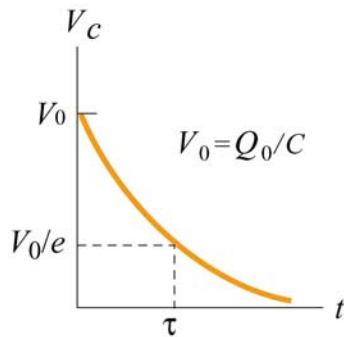
The voltage across the capacitor in a discharging *RC* circuit is given by

$$V_C(t) = \frac{q(t)}{C} = \left(\frac{Q_0}{C}\right)e^{-t/\tau}\,.$$

Figure 4 *RC* **circuit with discharging capacitor**

A graph of voltage across the capacitor as a function of time for the discharging capacitor is shown in Figure 5:

Figure 5 Voltage as a function of time for a discharging capacitor

The current also exponentially decays in the circuit as can be seen by differentiating the charge on the capacitor;

$$I(t) = -\frac{dq}{dt} = \left(\frac{Q_0}{RC}\right)e^{-t/\tau}. \tag{4.3}$$

This functional form is identical to the current found in Equation (4.2) and shown in Figure 3.

EXPERIMENTAL SETUP

A. AC/DC Electronics Lab Circuit Board

1. In this experiment we use the signal generator function of the 750 as a "battery" that turns on and off. The Signal Generator ports of the 750 Interface are the two ports on the right face of the Interface, labeled **OUTPUT**, as shown in Figure 6. Locate these ports on your 750 Interface.

Figure 6 The 750 Interface

2. Connect the banana plug patch cords from the "OUTPUT" ports of the 750 Interface to the banana jacks on the lower right corner of the AC/DC Electronics Lab circuit board (see locations D and E, at the lower right in Figure 7 below).
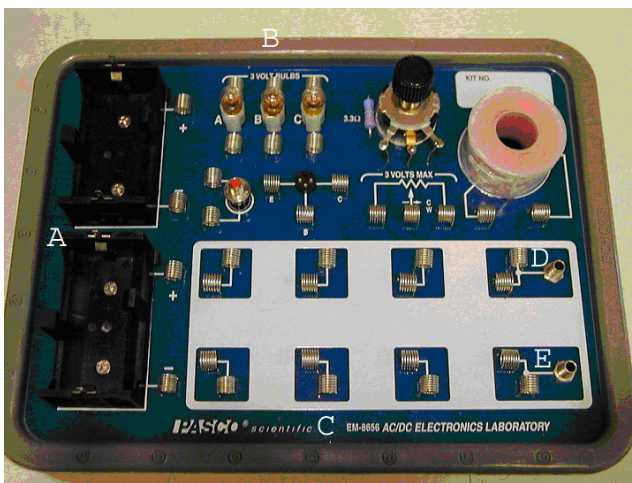

Figure 7 The AC/DC Electronics Lab Circuit Board

3. Place a 100-Ω resistor in the pair of springs nearest to the banana jacks at the lower right corner on the AC/DC Electronics Lab. The springs are connected by conductors to the jacks. The color-code for a 100-Ω resistor is brown-black-brown.

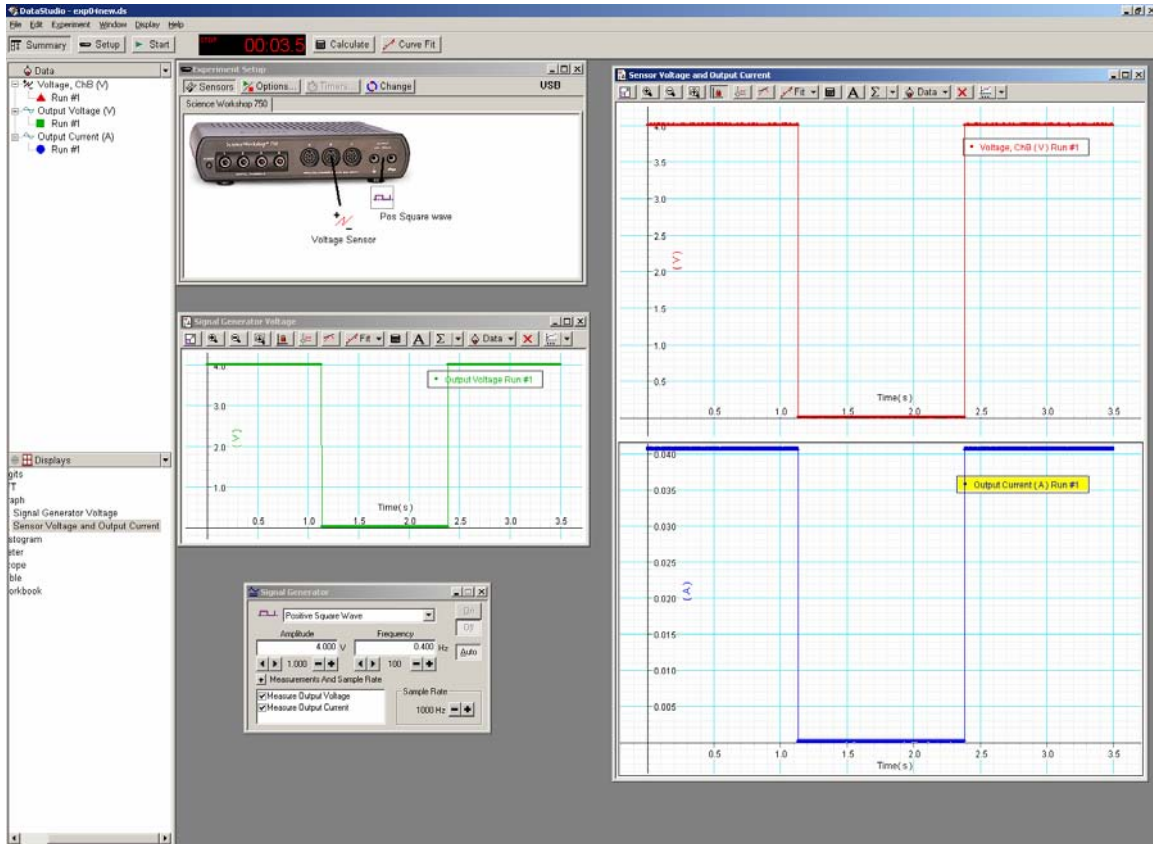## B. **Voltage Sensor** Setup:

The Voltage Sensor should be plugged into Analog Channel B of your 750, as shown in Figure 8.
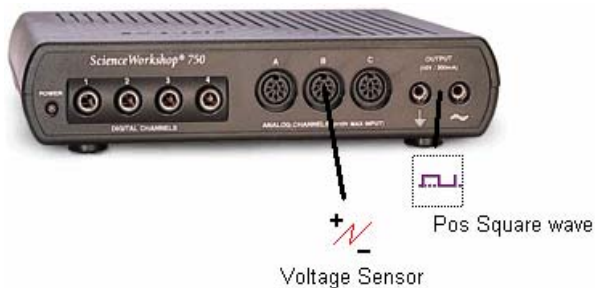
Figure 8 Voltage Sensor Setup

B. *DataStudio* File

Right click on the **exp04.ds** file from the website and download it to your desktop. Your file has an Experiment Setup display, a Signal Generator display, a Signal Generator Voltage graph display and a Sensor Voltage and Output Current graph display (see Figure 9).

Figure 9 *DataStudio* Activity display. We plot the output voltage from the signal generator in the graph on the left (in green) , the voltage sensor reading in the upper right panel (in red), and the output current in lower right panel (in blue).

Graphs: Here's how to set up the graphs above if you ever need to (it should already be set up for you here). Grab the Output Voltage icon in the Data window and drag it into the Graph icon. This will create the Signal Generator Voltage vs. Time graph. Grab the Voltage, ChB icon in the Data window and drag it into the Graph icon. This will create a Voltage Sensor vs. Time graph. Grab the Output Current (A) icon in the Data window and drag it into the Voltage, ChB graph icon. This will create a single display window with graphs of both the voltage sensor voltage and the output current.

Sampling Options: Click on the drop-down menu labeled Experiment on the top tool bar. In the Experiment menu, click on Set Sampling Option to open the Sampling Options dialog. Check that the Delay Choice is on None. Check that the Automatic

Stop choice is Time with 3.5 seconds in the window. If these options are not set in this manner, set them to these values.

C. Signal Generator:

We use the signal generator in this experiment as a "battery" that turns on and off in a step function fashion. To do this, in the Signal Generator dialog (Figure 10) we have chosen "Pos(itive) Square Wave Function."

The Amplitude has been adjusted to 4.000 V , the Frequency to 0.400 Hz and the Sampling Rate to 1000 Hz . We chose the output data that you will record by clicking the plus button (+) beside Measurements and Sample Rate on the Signal Generator dialog and clicking the appropriate Measure Output Voltage and Measure Output Current buttons.
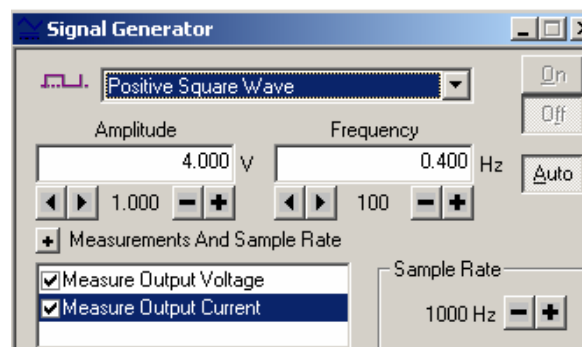

Figure 10 Signal Generator display

Part I: Ohm's Law--Measuring Voltage, Current, and Resistance

In this part of the experiment, you will assemble a circuit with resistors, and measure the voltage drops across various elements in the circuit, using the Positive Square Wave from the Signal Generator as a voltage source.

First, you should have a 100-Ω resistor in the pair of springs nearest to the banana jacks at the lower right corner on the AC/DC Electronics Lab. Place the leads for the voltage sensor in parallel with the 100-Ω resistor. We use the Measure Output Current feature of the Signal Generator to measure the current in this series circuit (this is an internal measurement made in the signal generator circuit, so we do not have to have an external ammeter in the circuit to measure the total current).

Press Start to begin taking data. Once the data has been recorded, scale the plots to fit the graph screens by clicking on the first icon on the left at the top of the Graph window (the Scale to Fit icon). Your *DataStudio* window should resemble that shown above (Figure 9).

Question 1 (*answer on your tear-sheet at the end*): What is the ratio of the maximum voltage measured by the voltage sensor to the maximum current measured in the circuit

when the voltage sensor is placed across your 100-Ω resistor? Is this ratio what you expect? Explain.

Now take the second 100-Ω resistor and put it in parallel with the first 100-Ω resistor. Leave the voltage sensor so that it is measuring the voltage across the two resistors in parallel. Press Start to begin taking data (if you want to get rid of the previous data run, go to Experiment on the top toolbar and choose Remove all Data runs).

Question 2 (*answer on your tear-sheet at the end*)**:** What is the ratio of the maximum voltage measured by the voltage sensor to the maximum current measured in the circuit when the voltage sensor is placed across your two 100-Ω resistors in parallel? Is this ratio what you expect? Explain.

Part II. Measuring Voltage and Current in an *RC* Circuit

In this part of the experiment, you will assemble an *RC* circuit, and apply a signal generator voltage (as above) in a manner that alternately charges the capacitor and allows the capacitor to discharge (the Square Wave output), as if we had a "battery" turning on and off. *DataStudio* will be used to determine the time constant of the circuits, both graphically and analytically.

The resistor/capacitor combination we use is two 100-Ω resistors in series with a 330-µF capacitor. On the Circuit Board (Figure 7) connect the 100-Ω resistors in series (color-code brown-black-brown) and in series with the capacitor, using the springs, so that the three elements form a closed loop; remember, for a series circuit the current is the same in each element.

We want to measure the voltage across the capacitor as well as the current in the circuit. In order to do this, we must connect the Voltage Sensor in *parallel* with the capacitor, with one clip at each end of the blue capacitor leads. Since we are dealing with series circuits, we again use the Measure Output Current feature of the Signal Generator to measure the current in this series circuit.

We use the same *DataStudio* file **exp04.ds** from the web page that we used in the first part of the experiment. If you want to get rid of old data runs, choose Experiment in the upper toolbar and Erase all Data runs. Press Start to begin taking data. Once the data has been recorded, scale the plots to fit the graph screens by clicking on the first icon on the left at the top of the Graph window (the Scale to Fit icon).

DATA ANALYSIS FOR *RC* CIRCUIT MEASUREMENTS

In this part of the experiment, you are asked to measure the time constant for an *RC* circuit as described above.

In setting up the apparatus, you should record data for two 100-Ω resistors in series with the 330-µF capacitor. You are asked to measure the time constant using both of the methods described below.

Method 1: The current in the discharging circuit with initial value I_0 at $t = 0$ decreases exponentially in time, $I(t) = I_0\ e^{-t/RC} = I_0\ e^{-t/\tau}$, where $\tau = RC$ is the **time constant,** as described above in Equation (4.3) and in the 8.02 *Course Notes*, **Section 7.6**.

You can determine the time constant τ graphically by measuring the current $I(t_1)$ at a fixed time t_1 and then finding the time $t_1 + \tau$ such that the current has the value

$$I(t_1 + \tau) = I(t_1)\,e^{-1} = 0.368\,I(t_1) \tag{4.4}$$
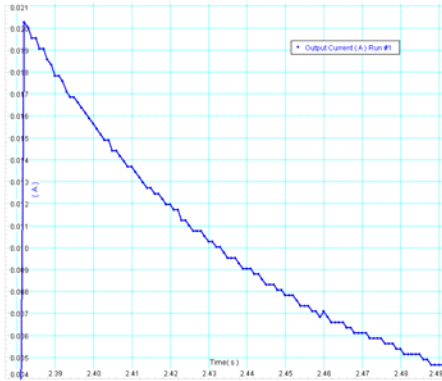
Figure 11 Current as a function of time in a discharging *RC* circuit.

Compare to Figure 3 above

In the current graph, enlarge the Graph window as desired by clicking and dragging anywhere on the edge of the graph window, or maximize the window. Click on the Zoom Select (fourth from the left) icon in the Graph icon bar and form a box around a region where there is exponential decay for the current. Click on Smart Tool (sixth from the left) icon. Move the crosshairs to any point (at some time t_1) on the exponentially decaying function (he Smart Tool display will become colored when the crosshairs are on a data point). Record the values of the time t_1 and the current I_1.

$$t_1 = \underline{\qquad} \qquad I_1 = \underline{\qquad} \qquad I(t_1 + \tau) = (0.368)\,I_1 = \underline{\qquad}$$

Multiply the current value (displayed in the Smart Tool feature) by $e^{-1} = 0.368$. (If you don't have a familiar calculator with you, the laptop should have this feature; go to Start at the lower left, and follow the prompts through Accessories and Calculator. The

DataStudio calculator can be used, but its use for basic arithmetic may seem somewhat cumbersome.) Use the Smart Tool to find the new time $t_1 + \tau$ such that the current is down by a factor of $e^{-1} = 0.368$. Of course, you won't find a data point with the exact value of $I(t_1 + \tau) = (0.368) I_1$; you may have to make an estimate, possibly from the graph.

Determine the time constant and record your value.

Questions 3 (*answer on your tear-sheet at the end*):

a. What is your measured value using Method 1 for the time constant for our circuit (two 100-Ω resistors in series with each other and with a 330-µF capacitor)?

b. What is the theoretical value of the time constant for your circuit?

c. How does your measured value compare to the theoretical value for your circuit? Express as a ratio, $\tau_{measured} / \tau_{theoretical}$.

**Method 2**: A second approach is to take the natural logarithm of the current, using the facts that $\ln\left(e^{-t/\tau}\right) = -t/\tau$ and $\ln(ab) = \ln a + \ln b$. This leads to

$$\ln\left(I(t)\right) = \ln\left(I_0\, e^{-t/\tau}\right) = \ln\left(I_0\right) + \ln\left(e^{-t/\tau}\right) = \ln\left(I_0\right) - t/\tau. \qquad (4.5)$$

Thus, the function $\ln\left(I(t)\right)$ is a linear function of time. The *y*-intercept of this graph is $\ln\left(I_0\right)$ and its slope is $slope = -1/\tau$. Thus, the time constant can be found from the slope of $\ln(I)$ versus time according to
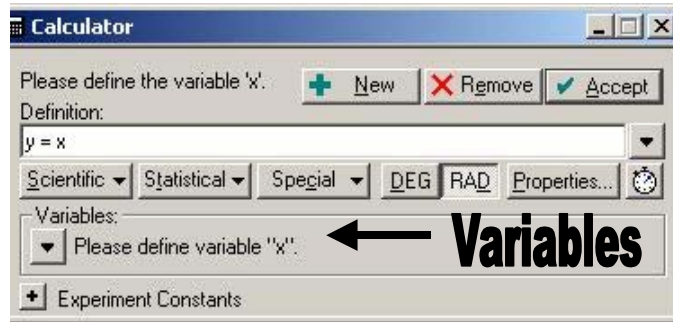
$$\tau = -1/slope \qquad (4.6)$$

We now want to calculate and plot $\ln(I)$ versus time, so that we can find this slope. This is a quantity which we do not measure, but which we can calculate given our current measurement. Click on Calculate from the Menu bar (see Figure 12).
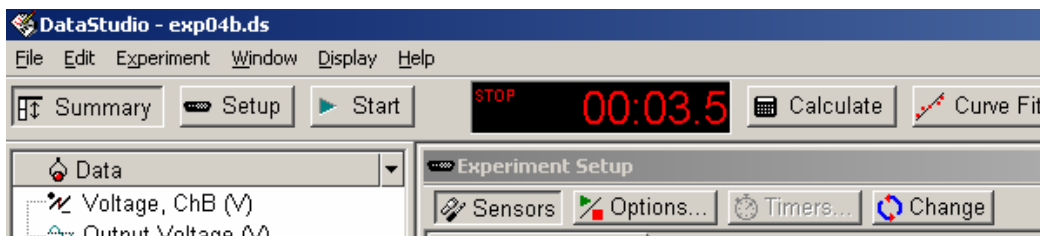
Figure 12 *DataStudio* Menu bar

A screen appears with y=x in the Definition field (see Figure 13).

Figure 13 Calculator window

In the Calculator window click New. Click on the Scientific button and scroll down and click on ln(x). Change the variable x in the Definition window to I (that's an uppercase "I" in the font used in *DataStudio*). Then click the Accept box in the upper right corner of the Calculator display. A Variables request now appears (see Figure 13), asking you to

       Please define variable "I"

When you click on the icon just to the left of Please define variable "I", a dropdown menu appears. Click on Data Measurement; a window appears titled Please Choose a Data Source (Figure 14). Click on Output Current [A] and OK.
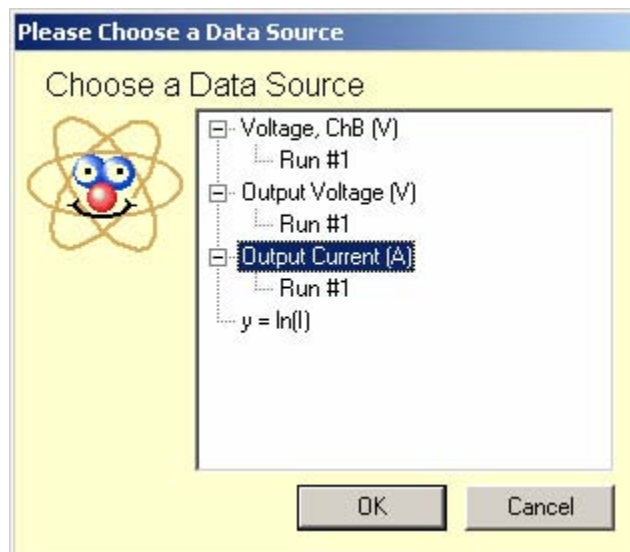

Figure 14 Please Choose a Data Source window

We have now defined the variable y =ln(I), and we want to plot it as a function of time. In the Data window, a calculator data type should have appeared with the text y =ln(I). Drag that calculator icon to the Graph icon in the Display window. A fairly complicated graph (most of which is no use to us, as the current is so small for most of the run) will appear (see Figure 15 below). Use the Zoom Select to isolate the small amount of data where the function is linear. You should see fluctuations in the data due to approximations associated with the sampling rate. Use the mouse to highlight a region of data where there are the smallest fluctuations. You can fit the highlighted data in that region using the Fit button (eighth icon from the left in your upper tool bar in the graph window). Click on that icon and scroll down and click to Linear Fit.

Record the value of the slope. Use your value of the slope to calculate the time constant.

Questions 4 (*answer on your tear-sheet at the end*):

a. What is your measured value using Method 2 for the time constant for our circuit (two 100-Ω resistors in series with a 330-μF capacitor)?

b. How does this Method 2 measured value compare to the theoretical value for your circuit? Express as a ratio, $\tau_{measured}/\tau_{theoretical}$.
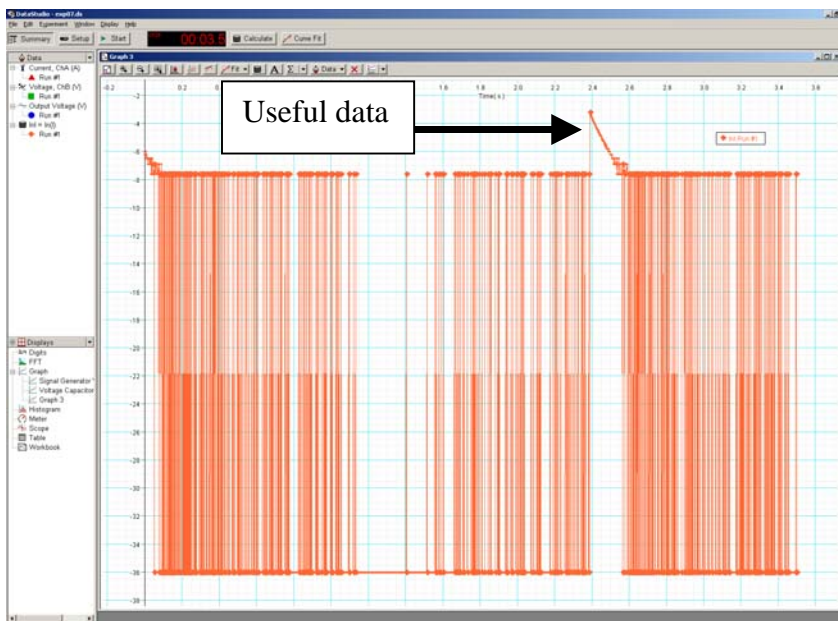
Figure 15
The $\ln(I(t))$ as a function of time plot of all the data. The region of useful data is indicated.

8.02   **Spring 2005**

Experiment Summary 4: Ohm's Law and *RC* Circuits

Group and Section _____ (**e.g. 10A, L02: Please Fill Out**)

Names _____

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Part I: Ohm's Law--Measuring Voltage, Current, and Resistance

Question 1: What is the ratio of the maximum voltage measured by the voltage sensor to the maximum current measured in the circuit when the voltage sensor is placed across your 100-Ω resistor? Is this what you expect? Explain.

Question 2: What is the ratio of the maximum voltage measured by the voltage sensor to the maximum current measured in the circuit when the voltage sensor is placed across your two 100-Ω resistors in parallel? Is this ratio what you expect? Explain.

Part II. Measuring Voltage and Current in an RC Circuit

Questions 3:
    a. What is your measured value using Method 1 for the time constant for our circuit (two 100-Ω resistors in series with a 330-μF capacitor)?

    b. What is the theoretical value of the time constant for your circuit?

    c. How does your measured value compare to the theoretical value for your circuit? Express as a ratio, $\tau_{measured} / \tau_{theoretical}$.

Questions 4:
    a. What is your measured value using Method 2 for the time constant for our circuit (two 100-Ω resistors in series with a 330-μF capacitor)?

    b. How does this Method 2 measured value compare to the theoretical value for your circuit? Express as a ratio, $\tau_{measured} / \tau_{theoretical}$.

IF YOU'D LIKE TO DO MORE …

Try a different combination of resistors, for example just use one 100 ohm resistor or use two 100 ohm resistors in parallel rather than in series. Use either one of the methods described above to determine the *RC* time constant with this new equivalent resistance.

Does your new time constant agree with what you expect theoretically? If your graphs get too crowded, you can eliminate previous runs; go to Experiment on the Menu bar and scroll down to eliminate all runs.