MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Problem Solving 5: Magnetic Force, Torque, and Magnetic Moments

OBJECTIVES

1. To start with the magnetic force on a moving charge q and derive the force on a wire segment carrying current I.

2. To calculate the torque on a rectangular loop of current-carrying wire sitting in an external magnetic field.

3. To define the magnetic dipole moment of a loop of current-carrying wire and write the torque on the loop in terms of that vector and the external magnetic field.

REFERENCE: Sections 8.3 – 8.4, 8.02 Course Notes.

Magnetic Force on a Straight Wire in an External Magnetic Field

Consider a straight wire of length L and cross sectional area A which carries a current I. The current consists of moving positive charge. The wire is placed in a constant external magnetic field $\vec{\mathbf{B}}_{ext}$. We want to calculate the force on this wire due to the magnetic force acting on the charge carriers in the wire. We take an infinitesimal current element of length dx in the external magnetic field $\vec{\mathbf{B}}_{ext}$ as shown in Figure 1.
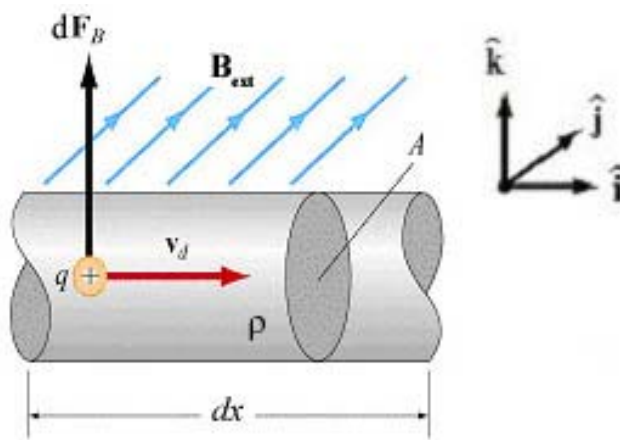

Figure 1: Infinitesimal current element of length dx in an external magnetic field.

In Figure 1, the charge carriers are assumed to be positive for simplicity and move to the right with a drift velocity $\vec{\mathbf{v}}_d$. Suppose the density of positive charge carriers is ρ coulombs/cubic meter.

Question 1 (*Write your answer on the tear-sheet at the end!*): What is the amount of charge dq in our line segment in terms of ρ, A, and dx?

Question 2 (*Write your answer on the tear-sheet at the end!*): In a time dt, all the charge in the element of length $dx = v_d \, dt$ will cross the area A at the end of the element. Find an expression for the total charge dq that crosses the area A in this time dt.

Question 3 (*Write your answer on the tear-sheet at the end!*): Find an expression for the current in the wire in terms of ρ, A, and v_d. Remember, the current in the wire is the amount of charge that flows pass a fixed point divided by the time it takes for that amount of charge to flow past that point.

Question 4 (*Write your answer on the tear-sheet at the end!*):

Introduce an infinitesimal current element vector $I d\vec{s}$ for a current carrying wire. This vector is tangent to the wire and points in the direction of the current, with $|d\vec{s}| = dx$. Since the charge carriers are moving in an external magnetic field, there is a infinitesimal magnetic force $d\vec{F}_B = dq\vec{v}_d \times \vec{B}_{ext}$. Using your answers from Question 1 and Question 3, you can show that $d\vec{F}_B = dq\vec{v}_d \times \vec{B}_{ext} = I d\vec{s} \times \vec{B}_{ext}$. In your answer on the tear-sheet, write down the algebraic steps you use to get this result.

The total force on the wire is the sum of all these infinitesimal forces. This is an integral over the wire,

$$\vec{F}_B = \int_{\text{wire}} d\vec{F}_B = \int_{\text{wire}} I d\vec{s} \times \vec{B}_{ext}$$

Force on a Rectangular Current Loop in a Uniform Magnetic Field

Given our result above, we now want to calculate the force on a loop of current-carrying wire sitting in a constant external magnetic field. Take a small rectangular square loop, with sides a and b, carrying a current I. The loop is placed in a uniform magnetic field, $\vec{B}_{ext}$ which is in the plane of the loop and points to the right (Figure 2). Let $\hat{\mathbf{n}}$ be the unit normal to the plane of the loop. We define the direction of $\hat{\mathbf{n}}$ to be pointing in a direction defined by your right thumb when you curl the fingers of your right hand in the direction of the current in the loop
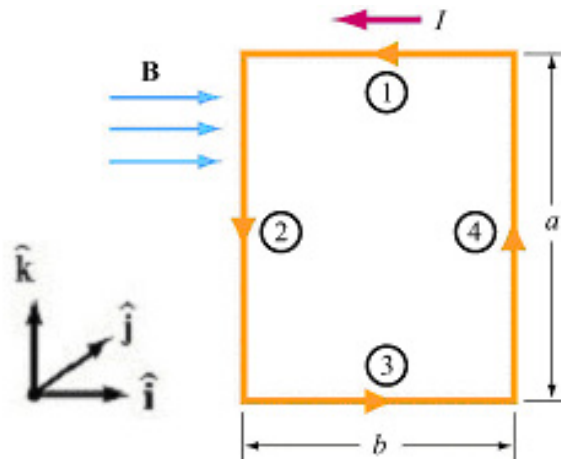
Figure 2: Current loop in an uniform magnetic field

In the coordinate system shown above, $\vec{\mathbf{B}}_{ext} = B\hat{\mathbf{i}}$. The sides aligned along the z-axis are length a. The normal to the loop as we defined it above is $\hat{\mathbf{n}} = -\hat{\mathbf{j}}$.

Question 5 (*Write your answer on the tear-sheet at the end!*): Using the formula $\vec{\mathbf{F}}_B = \int_{wire} Id\,\vec{\mathbf{s}} \times \vec{\mathbf{B}}_{ext}$, calculate the force on side 4 of the loop. Indicate both magnitude and direction.

Question 6 (*Write your answer on the tear-sheet at the end!*): Calculate the force on side 3 of the loop. Indicate both magnitude and direction.

Question 7 (*Write your answer on the tear-sheet at the end!*): Calculate is the force on side 2 of the loop.

Question 8 (*Write your answer on the tear-sheet at the end*!): Calculate the force on side 1 of the loop.

Question 9 (*Write your answer on the tear-sheet at the end*!): What is the total force on the loop (sum the forces on each side)?

Torque on a Rectangular Current Loop in a Uniform Magnetic Field:

In Figure 3a (next page) we repeat Figure 2, and in Figure 3b, we show the loop as seen looking up the $\hat{\mathbf{k}}$ axis (the $\hat{\mathbf{k}}$ vector is into the page in Figure 3b). In order to calculate the torque on side 4 about the center of the loop, it is sufficient to consider the entire force on side 4 as acting at the midpoint of the side. The torque on side 4 is given by

$$\vec{\boldsymbol{\tau}}_4 = \vec{\mathbf{r}}_4 \times \vec{\mathbf{F}}_4,$$

where $\vec{\mathbf{r}}_4$ is the vector from the center of the loop to the midpoint of the side and $\vec{\mathbf{F}}_4$ is the force we calculated in Question 5 above.

Question 10 (*Write your answer on the tear-sheet at the end*!): What is the torque $\vec{\boldsymbol{\tau}}_4$ about the center of the loop due to the force acting on side 4? Indicate its magnitude and direction.
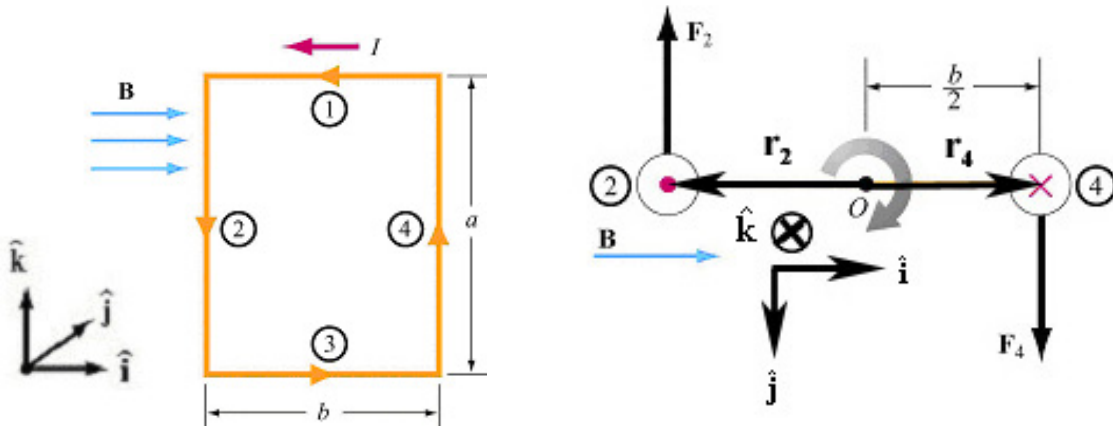
Figure 3: (a) A repeat of Figure 2; (b) The loop as seen looking up the $\hat{k}$ axis

Question 11 (*Write your answer on the tear-sheet at the end!*): What is the torque $\vec{\tau}_2$ about the center of the loop due to the force acting on side 2?

Question 12 (*Write your answer on the tear-sheet at the end!*): Why is the torque on sides 1 and 3 zero?

Question 13 (*Write your answer on the tear-sheet at the end!*): What is the total torque $\vec{\tau} = \vec{\tau}_4 + \vec{\tau}_2$ on the loop?

Question 14 (*Write your answer on the tear-sheet at the end!*): What effect will this torque have on the loop?

Magnetic Dipole Moment

The magnetic dipole moment vector $\vec{\boldsymbol{\mu}}$ of a planar current loop is defined as follows. For a planar loop, $\vec{\boldsymbol{\mu}}$ is defined in terms of the current I, the area A of the loop, and the unit normal $\hat{\mathbf{n}}$ to the plane of the loop,

$$\vec{\boldsymbol{\mu}} \equiv I\vec{\mathbf{A}} = IA\hat{\mathbf{n}}$$

The normal $\hat{\mathbf{n}}$ points in a direction defined by your right thumb when you curl the fingers of your right hand in the direction of the current in the loop.

The torque on a planar current loop or any magnetic dipole in a uniform magnetic field $\vec{\mathbf{B}}_{ext}$ can then be shown to be

$$\vec{\boldsymbol{\tau}}_{magnetic} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}_{ext} \, .$$

This result mirrors our earlier result for electric dipoles. When we placed an electric dipole $\vec{\mathbf{p}}$ in a uniform electric field $\vec{\mathbf{E}}_{ext}$, we found that the torque on the electric dipole was given by

$$\vec{\boldsymbol{\tau}}_{electric} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}_{ext} \, .$$

Question 15 (*Write your answer on the tear-sheet at the end*!): Calculate the torque on the planar current loop shown in Figure 2, using the definitions and expressions given above. Does this agree with the result you found in Question 13 above?

Problem: A square loop of wire, of length ℓ on each side, pivots about an axis AA' that corresponds to a horizontal side of the square, as shown in Figure 4. A magnetic field of magnitude B is directed vertically downward, and uniformly fills the region in the vicinity of the loop. The current I in the loop moves counterclockwise as viewed from above.
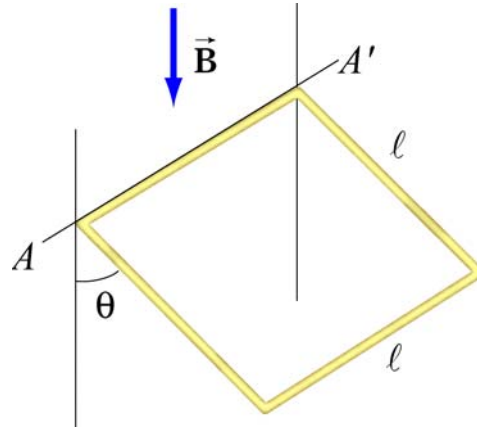

Figure 4: A current loop in an external field.

Question 16 (*Write your answer on the tear-sheet at the end!*): Calculate the magnitude of the torque on this loop of wire in terms of the quantities given, using our expressions above.

Question 17 (*Write your answer on the tear-sheet at the end!*): Does the sense of this torque make the coil pivot so that the angle θ increases or decreases?

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Tear off this page and turn it in at the end of class !!!!

Note:
Writing in the name of a student who is not present is a Committee on Discipline offense.

Problem Solving 5: Magnetic Force, Torque, and Magnetic Moments

Group _____ **(e.g. 6A Please Fill Out)**

Names _____

_____

_____

Question 1: What is the amount of charge dq in our line segment in terms of ρ, A, and dx?

Question 2: Find an expression for the total charge dq that crosses the area A in time dt.

Question 3: Find an expression for the current in the wire in terms of ρ, A, and v_d.

Question 4: Show that $d\vec{\mathbf{F}}_B = dq\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}_{\text{ext}} = Id\vec{\mathbf{s}} \times \vec{\mathbf{B}}_{\text{ext}}$. Write down the algebraic steps you use to get this result.

Question 5: Calculate the force on side 4 of the loop. _____

Question 6: Calculate the force on side 3 of the loop. _____

Question 7: Calculate is the force on side 2 of the loop. _____

Question 8: Calculate the force on side 1 of the loop. _____

Question 9: What is the total force on the loop (sum the forces on each side)? _____

Question 10: What is the torque $\vec{\tau}_4$ acting on side 4? _____

Question 11: What is the torque $\vec{\tau}_2$? _____

Question 12: Why is the torque on sides 1 and 3 zero? _____

Question 13: What is the total torque $\vec{\tau} = \vec{\tau}_4 + \vec{\tau}_2$ on the loop? _____

Question 14: What effect will this torque have on the loop? _____

Question 15: Calculate the torque on the planar current loop shown in Figure 2. Does this agree with the result you found in Question 13 above?

Question 16: Calculate the magnitude of the torque on the loop of wire in Figure 4 in terms of the quantities given, using our expressions above.

Question 17: Does the sense of this torque make the coil pivot so that the angle θ increases or decreases?