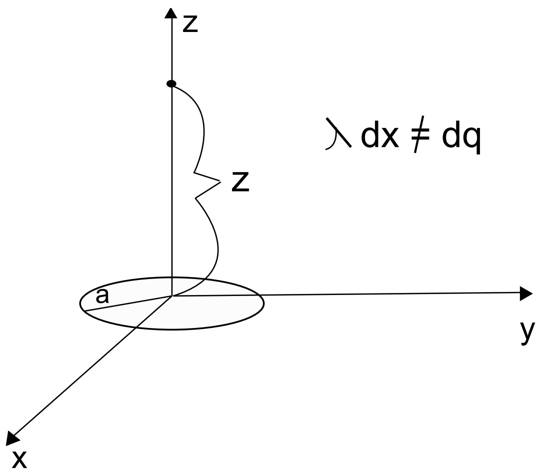
8.022 Lecture Notes Class 8 - 09/19/2006

What is the E-Field on the axis of a circular loop of a uniformly charged thin wire with total charge q?

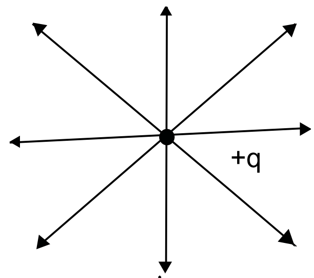


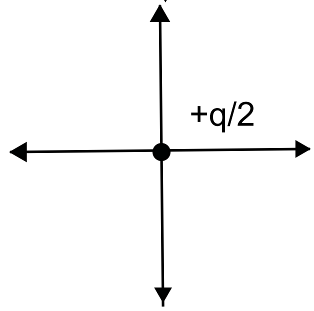
$$\begin{aligned} dE_z &= dE\cos\alpha \\ &= dE\frac{z}{\sqrt{a^2+z^2}} \end{aligned}$$

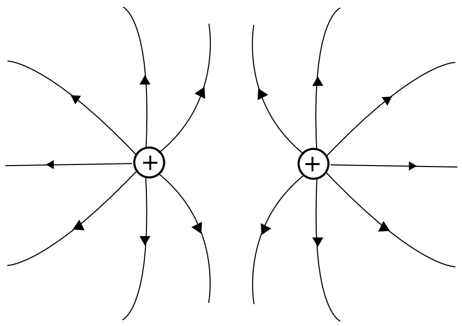
$$\begin{aligned} \vec{E}(z) &= \frac{1}{4\pi\epsilon_0}\oint_C \frac{dq}{r^3}\vec{r} \\ &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\frac{q}{2\pi a}\frac{(\cos\theta\hat{i}+\sin\theta\hat{j}+z\hat{k})}{(a^2+z^2)^{3/2}}r\,d\theta \\ &= \frac{qz}{4\pi\epsilon_0}\frac{\hat{k}}{z^2(a^2+z^2)^{1/2}} \\ &= \frac{q}{4\pi\epsilon_0}\cdot\frac{1}{z(a^2+z^2)^{1/2}} \end{aligned}$$

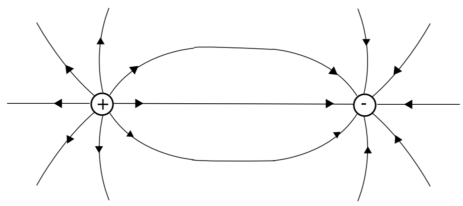
Field Lines

- Lines start and end on charges
- Number of lines proportional to charge
- Line density proportional to field strength
- $\frac{1}{r^2}$ in 3D space
- <u>Never</u> cross









Point Charge at Origin

$$\int_{Sphere} \vec{E} \cdot d\vec{a} = \int \frac{q}{4\pi\epsilon_0} \frac{\hat{r}}{r^2} (r^2 \sin\theta d\theta d\phi \hat{r}) \text{ where } d\vec{a} = \hat{r}(r^2 \sin\theta d\theta d\phi)$$

$$= \frac{q}{4\pi\epsilon_0} \hat{r} \cdot \hat{r} \int \sin\theta d\theta d\phi$$

$$= \frac{q}{\epsilon_0}$$

$$\Rightarrow \int \vec{E} \cdot d\vec{a} = \frac{q}{\epsilon_0}$$

$$E = \Sigma_i E_i$$

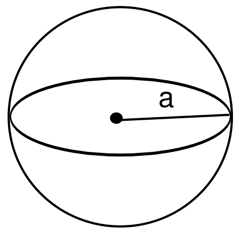
$$\int \vec{E} \cdot d\vec{a} = \Sigma_i \int \vec{E}_i \cdot d\vec{a} = \Sigma_i \frac{q_i}{\epsilon_0} = \frac{1}{\epsilon_0} Q_{enclosed}$$

$$\begin{aligned} \int_S \vec{E} d\vec{a} &= \int_V \vec{\nabla} \cdot \vec{E} d\tau \\ \frac{1}{\epsilon_0} Q_{enclosed} &= \quad \text{''''} \\ \frac{1}{\epsilon_0} \int_V \rho d\tau &= \int_V \vec{\nabla} \cdot \vec{E} d\tau \\ \frac{\rho}{\epsilon_0} &= \quad \vec{\nabla} \cdot \vec{E} \end{aligned}$$

Gaussian Surfaces and Symmetry

Spherical $\quad \vec{E} = \vec{E}(r)$ where r is the a distance.

$\vec{E}$ everywhere
Find $\vec{E}(r)$ for $r > a$
and $r < a$.



- $r > a$

$$\begin{aligned} \int \vec{E} \cdot d\vec{a} &= \tfrac{1}{\epsilon_0} Q_{enclosed} \\ E(r) \cdot \int d\vec{a} &= \quad \tfrac{Q}{\epsilon_0} \\ E(r) \cdot 4\pi r^2 &= \quad \tfrac{Q}{\epsilon_0} \\ E(r) &= \quad \tfrac{Q}{4\pi\epsilon_0 r^2} \end{aligned}$$

$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$$

- $r < a$

$$
\begin{aligned}
\int \vec{E} \cdot d\vec{a} &= \tfrac{1}{\epsilon_0} Q_{enclosed} \\
E(r) \cdot 4\pi r^2 &= \frac{Q}{\epsilon_0} \cdot \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} \\
E(r) &= \frac{Qr}{4\pi\epsilon_0 a^3} \\
\vec{E}(\vec{r}) &= \frac{Qr}{4\pi\epsilon_0 a^3}\hat{r}
\end{aligned}
$$