8.022 Lecture Notes Class 40 - 11/30/2006

Since $\epsilon_0\mu_0\vec{S} = p_{\text{em}}$

$$\vec{F} = \oint_S T \cdot d\vec{a} - \epsilon_0\mu_0\frac{d}{dt}\int_V \vec{S}d\tau = \frac{dP_{\text{mechanical}}}{dt}$$

$$\frac{dW}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}[\epsilon_0 E^2 + \frac{1}{\mu_0}B^2]d\tau - \int_S \frac{1}{\mu_0}\vec{E}\times\vec{B}d\vec{a}$$

First term is energy stored in fields and secon term is enrgy flowing in and out of region

$$\boxed{\begin{array}{c}\mathbf{P} = \text{total momentum}\\ p = \text{momentum density}\end{array}}$$

$$\vec{\mathbf{P}}_{\text{em}} = \mu_0\epsilon_0\int_V \vec{S}d\tau \qquad \text{momentum in E, B fields}$$

$$\oint T \cdot d\vec{a} \Longleftrightarrow \qquad \text{momentum flux through a surface}$$

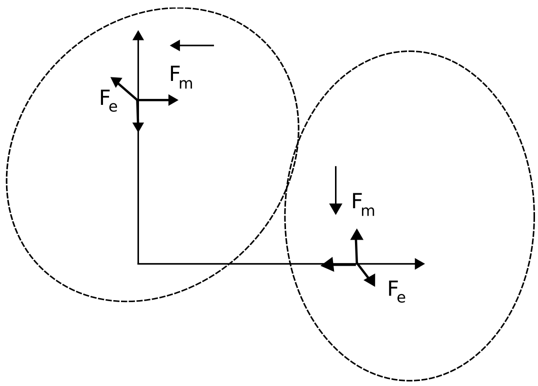
Conservation of Momentum for Electrodynamics

$$\boxed{\frac{\partial}{\partial t}(p_{\text{em}} + p_{\text{mechanical}}) = \vec{\nabla}\cdot T}$$

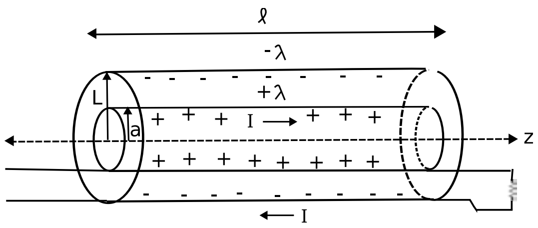
T_{ij} - momentum in i direction through a surface, normal in j-direction.

$$\vec{B} = \frac{\mu_0}{2\pi}\cdot\frac{I}{r}\hat{\phi} \qquad \vec{E} = \frac{1}{2\pi\epsilon_0}\cdot\frac{\lambda}{r}\hat{r}$$

$$\vec{S} = \frac{\lambda I}{4\pi^2\epsilon_0 r^2}\hat{z} \longrightarrow \text{ energy moving to the right}$$



Answer: field lines will "repel" - forces balanced out !



Find electromagnetic momentum (in fields)

$$\begin{aligned}\vec{\mathbf{P}}_{\mathrm{em}} &= \mu_0\epsilon_0 \int_V \vec{S} d\tau = \mu_0 \frac{\lambda I}{4\pi^2} \int_V \frac{1}{r^2} d\tau \\ &= \mu_0 \frac{\lambda I}{4\pi^2} \int \frac{1}{r^2} l \cdot 2\pi \cdot r dr \hat{z} \\ \vec{\mathbf{P}}_{\mathrm{em}} &= \frac{\mu_0 \lambda I l}{2\pi} \ln(\frac{b}{a}) \hat{z}\end{aligned}$$

But it's not moving. Well, $\vec{\mathbf{P}}_{\mathrm{em}}$ carried by current.

Relativity and Fun

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$p^\mu = \begin{pmatrix} \frac{E}{c} \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{in local time}$$

Minkowski force (τ is proper time)

$$K^\mu = \frac{dp^\mu}{d\tau} = \frac{q(\vec{E} + \vec{v} \times \vec{B})}{\sqrt{1 + \frac{v^2}{c^2}}}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ -\frac{E_x}{c} & 0 & B_z & -B_y \\ -\frac{E_y}{c} & -B_z & 0 & B_x \\ -\frac{E_z}{c} & B_y & -B_x & 0 \end{pmatrix}$$

$$\begin{bmatrix} 0 & E \\ -E & \epsilon_{\text{ijk}} B \end{bmatrix}$$

$$T = \begin{bmatrix} T_{00} & T_{\text{i0}} \\ T_{\text{0j}} & T_{\text{ij}} \end{bmatrix}$$

$$T^{\mu\nu} = \frac{1}{\mu_0} [F^{\mu p} F_p^\nu - \frac{1}{4} g^{\mu\nu} F^{p\sigma} F_{\text{p}\sigma}]$$

$$T^{00} = \frac{1}{\mu_0} [\frac{E^2}{2C^2} + \frac{B^2}{2}] = \frac{\epsilon_0}{2} E^2 + \frac{1}{2\epsilon_0} B^2$$

$$T^{0\mathrm{i}} = \frac{S_i}{C} = T^{\mathrm{i}0}$$

$$T = \begin{bmatrix} \text{energy} & \text{energy flux} \\ | & \text{momentum flux} \end{bmatrix}$$