8.022 Lecture Notes Class 25 - 10/30/2006

Biot-Savart

$$\vec{B}(\vec{x}) = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \hat{r}}{r^2}$$

Ampere's Law (like Gauss, but for $\vec{B}$)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \iint \vec{J} \cdot d\vec{a}$$

Stokes Theorem $\Downarrow$

$$\iint\!\!\oint \vec{\nabla} \times B da = \mu_0 \iint \vec{J} \cdot d\vec{a}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad : \text{Magnetostatics}$$
$$\vec{\nabla} \times \vec{E} = 0 \quad : \text{Electrostatics}$$

Boxed equations become Maxwell's equations!

$$\vec{B}(\vec{x}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r'}) \times (\hat{r} - \hat{r'})}{(r-r')^2} d^3r'$$

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \int \vec{\nabla}(\vec{J} \times \frac{(\hat{r} - \hat{r'})}{(r-r')^2} d^3r')$$

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \int 0 d^3r' = 0$$

$$\vec{\nabla}\cdot(\vec{J}\times\frac{(\hat{r}-\hat{r'})}{(r-r')^2}) = \frac{(\hat{r}-\hat{r'})}{(r-r')^2}(\vec{\nabla}_r\times\vec{J}(\vec{r'}))-\vec{J}\cdot(\nabla\times\frac{(\hat{r}-\hat{r'})}{(r-r')^2}\frac{(r-r')}{(r-r')^2}) \longrightarrow (\nabla_r)\times\frac{\hat{u}}{u^2}$$

(point charge) E, but $\vec{\nabla}\times\vec{E}=0$

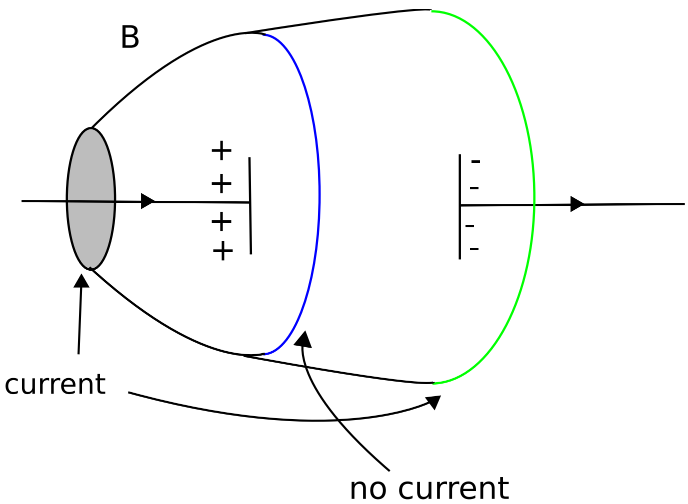
$$\begin{aligned} \vec{\nabla}\cdot\vec{B} = 0 \quad &: \text{Magnetostatics} \\ \vec{\nabla}\cdot\vec{E} = \tfrac{\rho}{\epsilon_0} \quad &: \text{Electrostatics} \end{aligned}$$

$$\begin{aligned} \vec{\nabla}\times\vec{B} &= \mu_0\vec{J} \\ \nabla\cdot(\vec{\nabla}\times\vec{B}) &= \mu_0(\vec{\nabla}\cdot\vec{J}) \\ 0 &= \vec{\nabla}\cdot\vec{J} = -\tfrac{\partial\rho}{\partial t} \end{aligned}$$

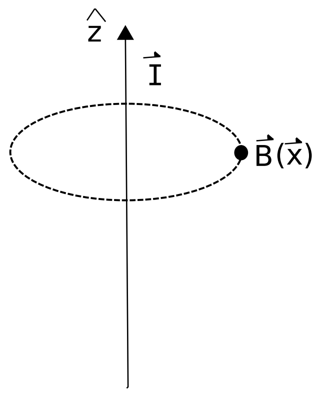
$\Rightarrow -\frac{\partial\rho}{\partial t}=0$ is from Magnetostatics : assumes constant current always
But current can't do that infinitely , so adjust

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}+\mu_0\vec{J}_d$$

J_d is displacement current <u>NOT</u> a real current



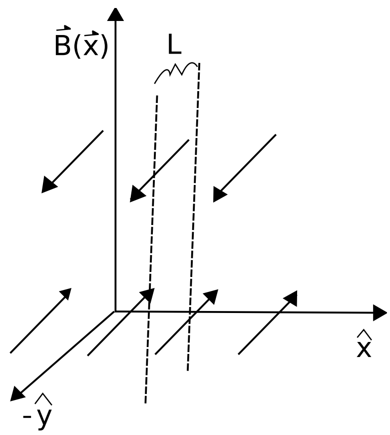
("We'll there's something where the current is displaced, so we'll call it displacement current")



Find $\vec{B}$ at $\vec{x}$ (Use's Ampere's)

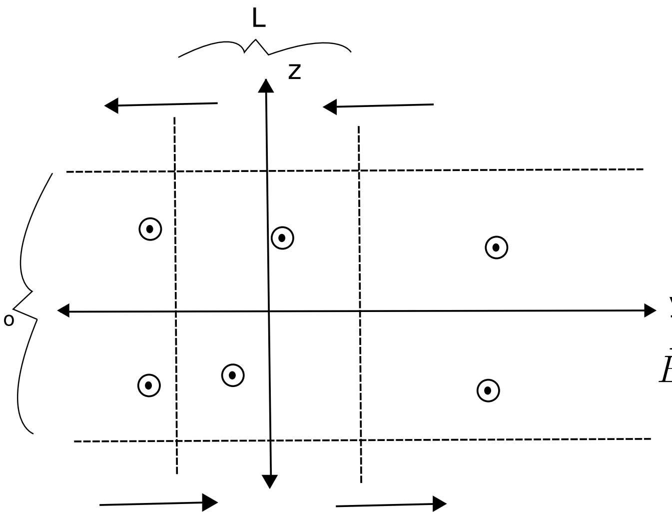
$$\int \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}} \quad \text{Ampere's}$$

$$\vec{B}(\vec{x}) = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r}\hat{\theta}$$



Current in x-y plane
Find $\vec{B}(\vec{x})$

$$\begin{aligned} \int \vec{B} \cdot d\vec{l} &= \quad \mu_0 I_{enc} \\ \hat{y}B \cdot \cdot (2L) &= \quad \mu_0 \cdot k \cdot L \\ \vec{B}(\vec{x}) &= \tfrac{\mu_0 k}{2}\hat{y} \quad z > 0 \end{aligned}$$



$$\int \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

Outside:

$$\vec{B} \cdot 2L = \mu_0 \cdot L 2 z_0 J_0$$

$$\vec{B} = \pm \mu_0 z_0 J_0 \hat{y} \quad (|z| > z_0)$$

Inside:

$$\vec{B} \cdot 2L = \mu_0 L \cdot 2z \cdot J_0$$

$$B = \pm \mu_0 z J_0 \quad (|z| < z_0)$$

$$\vec{J} = \begin{cases} J_0 \hat{x}, |z| < z_0 \\ 0, else \end{cases}$$

Graph: $\vec{B}(z)$