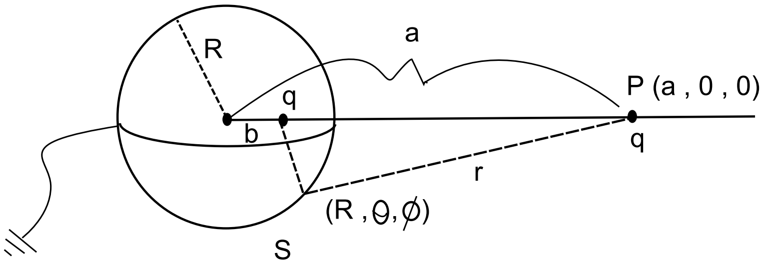
8.022 Lecture Notes Class 15 - 10/03/2006



Find $V(\vec{x})$, $|\vec{x}| \geq R$

$V(r, \theta, \phi)|_{r=R} = 0$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}(\frac{q}{r} + \frac{q'}{r'}) = 0 \quad \text{(on sphere)}$$

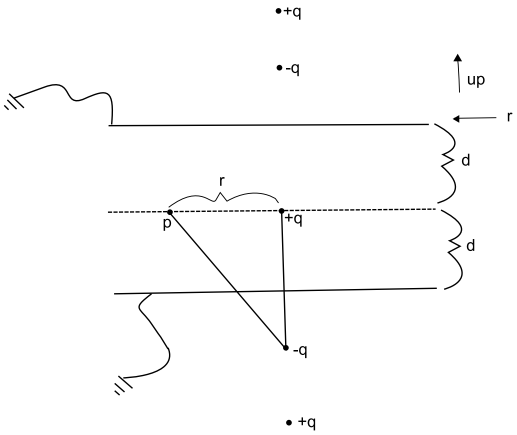
$b = \frac{R^2}{a}$

• Do it for point on sphere between a and b.

$$\begin{aligned} V(R,0,0) &= \frac{1}{4\pi\epsilon_0}(\frac{q}{a-R} + \frac{-q'}{\frac{-R^2}{a}+R}) \\ &= \frac{1}{4\pi\epsilon_0}(\frac{-qR^2}{a} + \frac{Ra}{a} - \frac{q'a^2-Raq'}{a}) \\ \frac{qR^2}{a} - \frac{Ra}{a} &= \frac{Raq'}{a} - \frac{q'a^2}{a} \\ qR^2 - Ra &= Raq' - q'a^2 \\ &= q'(Ra - a^2) \end{aligned}$$

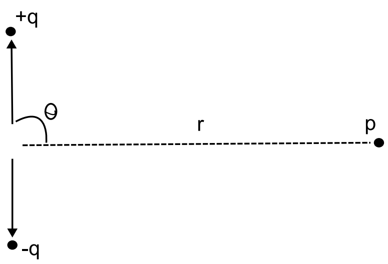
$$\begin{aligned} q' &= \frac{qR(Ra)}{(Ra)a} \\ q' &= \frac{qR}{a} \end{aligned}$$

Parallel Plate Capacitor



$$V(r)|_{up=0}\frac{q}{4\pi\epsilon_0}(\frac{1}{r}+\Sigma_{k=1}^{\infty}\frac{z(-1)^k}{\sqrt{r^2+4d^2k^2}})$$

Electric Dipole



$$\begin{aligned} V(\vec{r}) &= \frac{1}{4\pi\epsilon_0}(\frac{q}{r_+}+\frac{-q}{r_-}) \\ r_\pm^2 &= r^2+(\frac{d}{2})^2 \mp rd\cos\theta \quad \text{(Law of Cosines)} \\ &= r^2(1+\frac{1}{r^2}(\frac{d}{2})^2 \mp \frac{1}{r}d\cos\theta) \\ &= r^2(1+(\frac{d}{r})^2\frac{1}{4}+\frac{-d}{r}\cos\theta), r >> d \end{aligned}$$

$$
\begin{aligned}
\frac{1}{r_\pm} &= \tfrac{1}{r}(1 + \tfrac{-d}{r}\cos\theta)^{-1/2} \\
&\approx \tfrac{1}{r}(1 + \pm\tfrac{d}{2r}\cos\theta) \quad \text{(Taylor series for } \sqrt{1+\tfrac{a}{x}}) \\
\frac{1}{r_+} - \frac{1}{r_-} &= \tfrac{1}{r}(1 + \tfrac{d}{2r}\cos\theta - 1 + \tfrac{d}{2r}\cos\theta) \\
V(\vec{r}) &= \tfrac{q}{4\pi\epsilon_0}(\tfrac{1}{r}\cdot\tfrac{d}{r}\cos\theta) \\
V(\vec{r}) &= \tfrac{qd}{4\pi\epsilon_0 r^2}\cos\theta
\end{aligned}
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