8.022 (E&M) – Lecture 8

Topics:

- Electromotive force
- Circuits and Kirchhoff's rules

Quiz 1: thoughts

- Average: 59, RMS: 16
  - Last year average: 64 → test slightly harder than average
  - Problem 1 had some subtleties + math looked scary
- When to worry
  - grade < 40: very serious danger, TEAL is strongly recommended
  - 40<grade<50: something is wrong, more work/changes needed
- The bottom line: 8.022 is a hard class
  - Fun (and useful) only if you are ready for it
  - If not, you may be better off with TEAL
- Prof. Kats and I are available for discussion/advise
  - Bring exam and psets with you
- Exams will be posted on the MIT server today at 12:30 PM

Last time

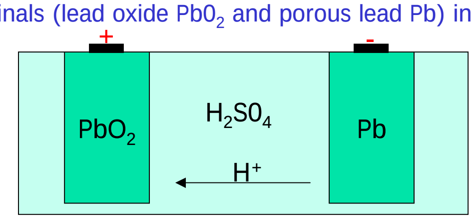
- Electric current I: $I = \frac{dQ}{dt}$
- Electric current density J: $\vec{J} \equiv qn\vec{u} \equiv \rho\vec{u}$
- Continuity equation: $\nabla \bullet \vec{J} + \frac{\partial \rho}{\partial t} = 0$
- Ohm's law:
  - Microscopic: $\vec{J} = \sigma \vec{E}$ Macroscopic: $V = IR$
    - σ=conductivity $\sigma = \frac{nq^2\tau}{m}$
    - R=resistance $\mathrm{R} \equiv \frac{L}{\sigma A}$

EMF: Electromotive force

- What makes charges flow in circuits?
  - Potential difference ΔV
  - Source of charges
- This is what the EMF provides
  - NB: EMF=Electromotive force but it's <u>not</u> a force!!!
- Example of EMF: battery
  - Device that maintains separation of charges between 2 electrodes
  - Current flows inside via electrochemical reactions that produce ΔV

Car Battery

- Two terminals (lead oxide PbO_2 and porous lead Pb) in sulfuric acid (H_2SO_4)



- When immersed in acid, Pb provides free electrons:

$$Pb + HSO_4^- \rightarrow PbSO_4 + H^+ + 2e^-$$

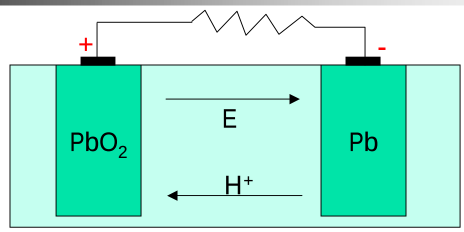
- At the lead oxide electron, this reaction is energetically favored:

$$PbO_2 + 3H^+ + HSO_4^- + 2e^- \rightarrow PbSO_4 + 2\,H_2O$$

- If it is possible for both e^- and H^+ to travel from one terminal to the other:

$$Pb + PbO_2 + H_2SO_4 \rightarrow 2PbSO_4 + 2\,H_2O \qquad +4.4\text{ eV}$$

Car Battery (2)



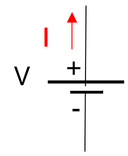
- When terminals are not connected: no flow of e-
- **E** in battery does not allow flow of H+ → inhibits reaction
- When terminals are connected: electrons start flowing freely
- Electric field is reduced → H^+ can flow → reaction occurs

EMF of battery: ϕ(+ terminal) – ϕ(- terminal): ΔV available to drive circuit

$$EMF = \int_{-\text{ terminal}}^{+\text{ terminal}} \vec{E} \bullet d\vec{s}$$

Convention

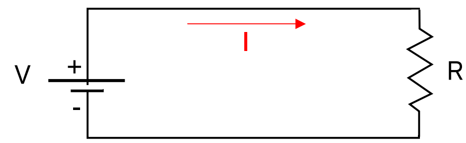
- We indicate EMF with this symbol:
  - Long side: + terminal
  - Short side: - terminal



- The current flows from + to –
  - Counterintuitive if you think about it in terms of electrons…

Kirchhoff' second rule

- Close a battery on a resistor: simplest circuit!



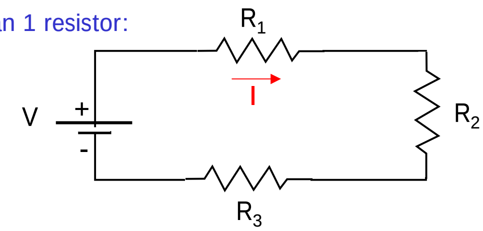
- How much current flows in the circuit? Ohm's law: $I = \frac{V}{R}$
- When the current flows in a resistor there is a voltage drop $\Delta V = -IR$

Kirchhoff's second law:
Around any closed loops, the sum of EMF and potential drops is 0

- Equivalent to say that Electrostatic filed is conservative: $\oint \vec{E} \bullet d\vec{s} = 0$

Solving circuits

- If we have more than 1 resistor:



- Solve the circuit: determine currents and voltages everywhere
- What we know:
  - Current flowing in the circuit must be the same everywhere, or Q would accumulate somewhere
  - Voltage drop in the i^{th} resistor: $\Delta V_i = -IR_i$
  - Second Kirchhoff rule: $V - \sum_i V_i = 0$

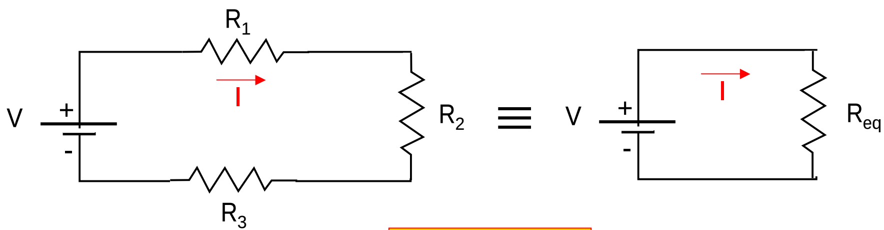
$$V - \sum_{i=1,3} V_i = V - I \sum_{i=1,3} R_i = 0 \quad \Rightarrow \quad I = \frac{V}{R1 + R2 + R3}$$

Resistors in series

- We implicitly derived an important result. We wrote:

$$V - \sum_{i=1,3} V_i = V - I \sum_{i=1,3} R_i = 0 \quad \Rightarrow \quad I = \frac{V}{R_1 + R_2 + R_3}$$

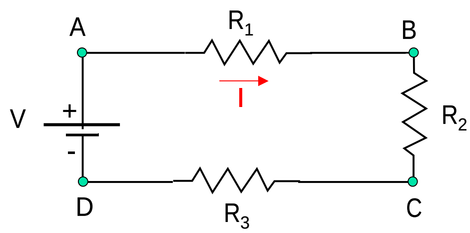
- What does it mean? Same current flowing in these two circuits:



$$R_{eq} = \sum_i R_i$$

Solving circuits

- Solve the circuit: determine currents and voltages everywhere



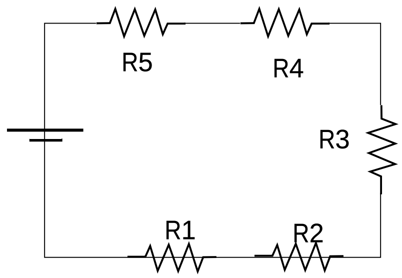
- Calculate V_{AB}, V_{BC}, V_{CD}, V_{DA}, V_{AC}, ...

$$V_{AB} = IR_1 = \frac{VR_1}{R_1 + R_2 + R_3}; \quad V_{BC} = IR_2 = \frac{VR_2}{R_1 + R_2 + R_3}$$

$$V_{CD} = IR_3 = \frac{VR_3}{R_1 + R_2 + R_3}; \quad V_{AC} = I(R_2 + R_1) = \frac{V(R_1 + R_2)}{R_1 + R_2 + R_3}$$

Application (F13)

- What is the resistance of electrical components?



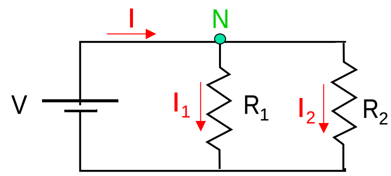
Elements of the circuit:

- Saline solution
- Resistor
- Diod
- light bulb
-Fluoreschent light

- How to measure knowing the current = 135 mV?
- You are given a voltmeter!

Kirchhoff's first rule

- Let's now connect resistors in parallel:



- At the node N the current I divides up into 2 pieces: I_1 and I_2

<u>Kirchhoff's first law:</u>
At any node, sum of the currents in = sum of the currents out

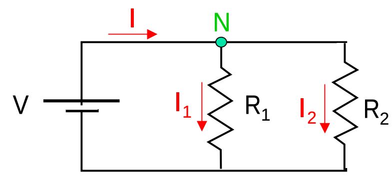
- In other words: there is no accumulation of charges in the circuit

Kirchhoff's first rule: application

- Solve the circuit:

$$V = I_1 R_1 \quad \Rightarrow \quad I_1 = \frac{V}{R_1}$$

$$V = I_2 R_2 \quad \Rightarrow \quad I_2 = \frac{V}{R_2}$$



- Apply Kirchhoff's first law: $I = I_1 + I_2$

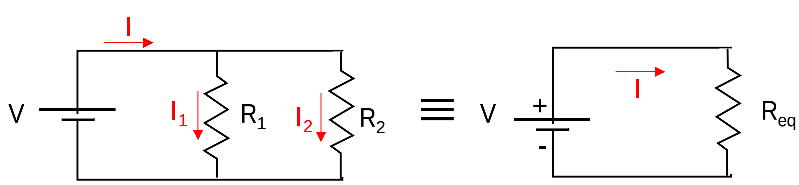
$$I = I_1 + I_2 = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Resistors in parallel

- Again, we are learning something important. We said:

$$I = I_1 + I_2 = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

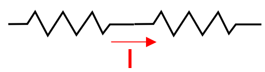
- What does it mean?



$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

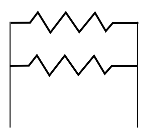
Resistors in parallel vs. in series

- Resistors in series:
  - The current flowing is one → add resistors → make the path harder → I decreases → R_{eq} increases → R_{eq} larger than any single resistor



$$R_{eq} = \sum_i R_i$$

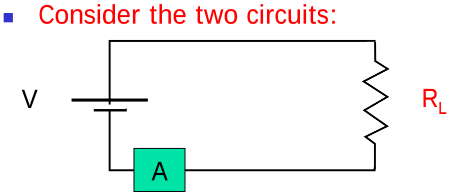
- Resistors in parallel:
  - The current flows is many resistors → add resistors → make path easier → I increases → R_{eq} is smaller than any single resistor



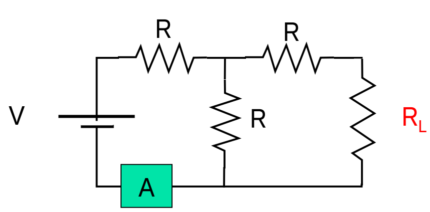
$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

Application (F12)

- Consider the two circuits:



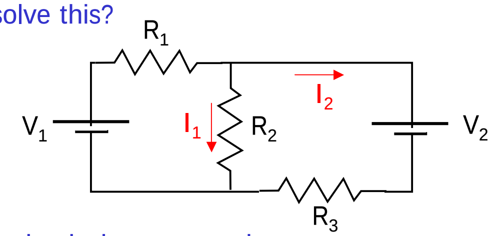
V= 1.5 V
R_L = 1580 Ω
Ammeter reading: 0.94 mA



R_L = 1580 Ω
R = 912 Ω
Ammeter reading: ?? mA

Slightly harder circuits

- How do we solve this?



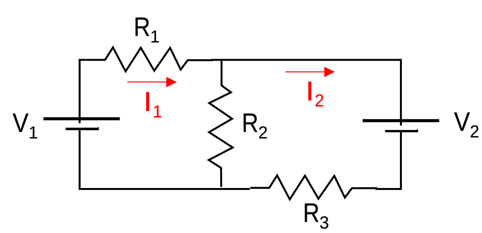
- Reducing the circuit does not work:
  - Series and parallels won't work
    - Because of second EMF
- But Kirchhoff still holds so:
  - Apply First Kirchhoff law to each node
  - Apply Second Kirchhoff law to each loop

Slightly harder circuits (2)

- Solution:
  - Left loop:
    - $V_1 - I_1R_1 - (I_1 - I_2)R_2 = 0$
  - Right loop:
    - $-V_2 - I_2R_3 - (I_2 - I_1)R_2 = 0$
  - Node:
    - I(in R2)=I1-I2
- Solving the system:

$$I1 = \frac{V1R3 + (V1 - V2)R2}{R1R2 + R1R3 + R2R3}$$

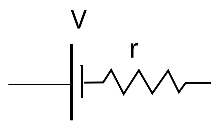
$$I2 = \frac{(V1 - V2)R2 + V2R1}{R1R2 + R1R3 + R2R3}$$



How to go through a loop?
- Assign current direction (arbitrary)
- Choose a path (clockwise or ACW)
- EMF: >0 when -→+ and <0 when +→-
- V always drops on R

Internal resistance

- When battery delivers current to circuit there is a flow of current in the battery itself.
  - In previous example this current comes from flux of H^+ ions
- The chemical reaction will dissipate energy, battery gets hot, energy that could have gone in the circuit is lost
  - This is equivalent to having a resistor r inside the battery:



- Corollary:
  There is a max current the battery can generate: I_{max}=V/r
- Homework:
  Short a little battery with a wire and see how hot it gets: careful! ☺

Power dissipated in resistors

- When current flows in circuit, it moves charges through ΔV → work
- Work done to drive a charge dq through a potential difference V:

$$dW = Vdq$$

- If work is done in time dt → the power dissipated is:

$$P = \frac{dW}{dt} = V\frac{dq}{dt} = VI$$

- When Ohm's law hold:

$$\boxed{P = VI = RI^2}$$

- Units: [P]=[Energy]/[time]
  - cgs: erg/s
  - SI: J/s

Power is important: it's what does work in a circuit: how much light is produced, how much heath, etc.

Dependence of R on T (F16)

- Ohm's law tells us that V=RI
- This is valid in any resistor
- Does it mean that given a voltage I is constant over time?
- Not necessarity!
  - When I goes through R it dissipates power=RI^2
  - R=f(T) → R is not constant while resistor heats up!
- Last time: increase T → increase R → decrease I
- Pulse a light bulb with a saw tooth voltage:
  What do you expect to observe?
  As the filament warms up, R changes
  → the slope of I vs V changes over time

Summary and outlook

- Today:
  - First look at circuits: Kirchhoff's laws
  - Resistors in series and parallel
  - Power dissipated in resistors
- Next time:
  - RC circuits and evolution of currents over time
- Remember: come and talk to us today if test < 50
  - Make appointment by email is safer
  - Add date is THIS FRI!!!