Massachusetts Institute of technology
Department of Physics
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Quiz #2: Formula sheet

- **Work on the problems you know how to solve first!**
- Exam is **closed book** and **closed notes**. Useful math formulae are provided below.
- No calculators will be needed.

Potential: $\phi(a) - \phi(b) = -\int_b^a \vec{E} \cdot d\vec{s}$

Energy of E: The energy of an electrostatic configuration $U = \frac{1}{2}\int_V \rho\phi dV = \frac{1}{8\pi}\int E^2 dV$.

Pressure: A layer of surface charge density σ exerts a pressure $P = 2\pi\sigma^2$.

Current density: $\vec{J} = \rho\vec{v}$ **.**

Current: $I = dQ/dt = \int_S \vec{J} \cdot d\vec{a}$ (I is the current through surface S).

Continuity: $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$

Ohm's law: $\vec{J} = \sigma_c \vec{E}$ (microscopic form): V=IR (macroscopic form)

Capacitance: Q=CV. **Energy stored in capacitor:** $U_C = \frac{Q^2}{2C} = \frac{1}{2}CV^2$

Lorentz force: $\vec{F} = q\vec{E} + q\frac{\vec{v}}{c} \times \vec{B}$

Magnetic force on current: $\vec{F} = \frac{I}{c} d\vec{l} \times \vec{B}$**;** or $\vec{F}/L = \frac{\vec{I}}{c} \times \vec{B}$

Vector potential: $\vec{B} = \nabla \times \vec{A}$; $\vec{A} = \frac{I}{c}\int \frac{d\vec{l}}{r}$

Biot-Savart law: $d\vec{B} = Id\vec{l} \times \hat{r}/(cr^2)$

Maxwell's equations in differential form (so far!!!):

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad \text{(Gauss's law)}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \quad \text{(Faraday's law)}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J} \text{ (Ampere's law)}$$

Maxwell's equations in integral form (so far):

$\int_S \vec{E}\cdot d\vec{a} = 4\pi Q$ (Gauss's law. Q is charge enclosed by surface S)

$\int_C \vec{E}\cdot d\vec{s} = -\frac{1}{c}\frac{\partial \phi_B}{\partial t}$ =e.m.f. (Faraday's law. ϕ_B is $\vec{B}$ flux through surface bounded by C.)

$\int_C \vec{B}\cdot d\vec{s} = \frac{4\pi}{c}$ I (Ampere's law. I is current enclosed by contour C.)

Self inductance: $\varepsilon = -LdI/dt$

Mutual Inductance: $\mathcal{E}_1 = -M_{12}dI_2/dt$; $\mathcal{E}_2 = -M_{21}dI_1/dt$; $M_{12} = M_{21}$

Magnetic energy: $U = \frac{1}{8\pi}\int B^2 dV$

Energy stored in an inductor: $U_L = \frac{1}{2}LI^2$

Time dilation: Moving clocks run slow: $\Delta t_{stationary} = \gamma \Delta t_{moving}$

Length contraction: Moving rulers are shortened: $L_{stationary} = L_{moving}/\gamma$

Transformation of fields: $||$ denotes parallel to $\vec{v}$, $\perp$ denotes perpendicular to $\vec{v}$

$$\vec{E}'_{||} = \vec{E}_{||} \qquad \vec{E}'_{\perp} = \gamma(\vec{E}_{\perp} + \frac{\vec{v}}{c}\times\vec{B}_{\perp})$$

$$\vec{B}'_{||} = \vec{B}_{||} \qquad \vec{B}'_{\perp} = \gamma(\vec{B}_{\perp} - \frac{\vec{v}}{c}\times\vec{E}_{\perp})$$

Useful Math

Cartesian.

Gradient: $\nabla t = \frac{\partial t}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial t}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial t}{\partial z}\hat{\boldsymbol{z}}$

Divergence: $\nabla \cdot \vec{\mathbf{v}} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \vec{\boldsymbol{v}} = (\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z})\hat{\boldsymbol{x}} + (\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x})\hat{\boldsymbol{y}} + (\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y})\hat{\boldsymbol{z}}$

Laplacian: $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

Spherical.

Gradient: $\nabla t = \frac{\partial t}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\varphi}}$

Divergence: $\nabla \cdot \vec{\mathbf{v}} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$

Curl:

$$\nabla \times \vec{\boldsymbol{v}} = \frac{1}{r\sin\theta}[\frac{\partial(\sin\theta v_\phi)}{\partial\theta} - \frac{\partial v_\theta}{\partial\phi}]\hat{\boldsymbol{r}} + \frac{1}{r}[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial(r v_\phi)}{\partial r}]\hat{\boldsymbol{\theta}} + \frac{1}{r}[\frac{\partial(r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial\theta}]\hat{\boldsymbol{\varphi}}$$

Laplacian: $\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial t}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$

Cylindrical.

Gradient: $\nabla t = \frac{\partial t}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\varphi}} + \frac{\partial t}{\partial z}\hat{\boldsymbol{z}}$

Divergence: $\nabla \cdot \vec{\mathbf{v}} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho v_\rho) + \frac{1}{\rho}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \vec{\mathbf{v}} = [\frac{1}{\rho}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}]\hat{\boldsymbol{\rho}} + [\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial\rho}]\hat{\boldsymbol{\varphi}} + \frac{1}{\rho}[\frac{\partial(\rho v_\phi)}{\partial\rho} - \frac{\partial v_\rho}{\partial\theta}]\hat{\mathbf{z}}$

Laplacian: $\nabla^2 t = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial t}{\partial\rho}) + \frac{1}{\rho^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$

Binomial expansion:

$$(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} \pm \cdots (x^2 < 1); (1 \pm x)^{-n} = 1 \mp \frac{nx}{1!} + \frac{n(n+1)x^2}{2!} \mp \cdots (x^2 < 1)$$

Stokes' theorem: $\oint_C \vec{F} \cdot d\vec{s} = \int_S \mathrm{curl}\,\vec{F} \cdot d\vec{A}$

Gauss' theorem: $\oint_S \vec{F} \cdot d\vec{A} = \int_V \mathrm{div}\,\vec{F} dV$