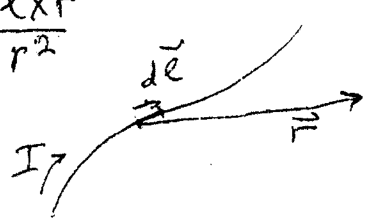
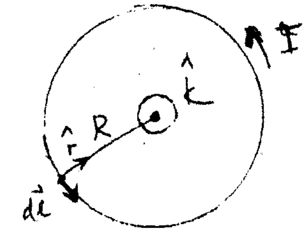
Fall 2002

Solutions to practice quiz for week #9

- Biot & Savart: $d\vec{B} = \frac{I}{c}\frac{d\vec{l}\times\hat{r}}{r^2}$



Apply to a circle current:



$$\vec{B}_0 = \int_{\text{on wire}} d\vec{B}_0 = \frac{I}{c}\int_{\text{on wire}}\frac{dl}{R^2}\hat{k}$$

$$= \frac{I}{cR^2}\cdot 2\pi R\,\hat{k} = \frac{2\pi I}{cR}\hat{k}$$

Calculate the contribution to B_0 of an arc θ:

$$\vec{B} = \frac{I}{c}\int_0^{\theta}\frac{dl\,R}{R^3}\hat{k} = \frac{I}{cR^2}\,\theta\cdot R\hat{k} = \theta\frac{I}{cR}\hat{k}$$

- Two arcs, one of radius R and angular opening θ and another one of radius $R+r$ and angular opening $2\pi-\theta$ contribute to the field at the center:

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\theta I}{cR}\hat{k} + \frac{(2\pi-\theta)I}{c(R+r)}\hat{k} = \frac{2\pi R + r\theta}{cR(R+r)}I\hat{k}$$

- Clearly as $r\to 0$. $\vec{B}\to\frac{2\pi R I\hat{k}}{cR^2} = \frac{2\pi I\hat{k}}{cR}$

the field at the center of a circular current