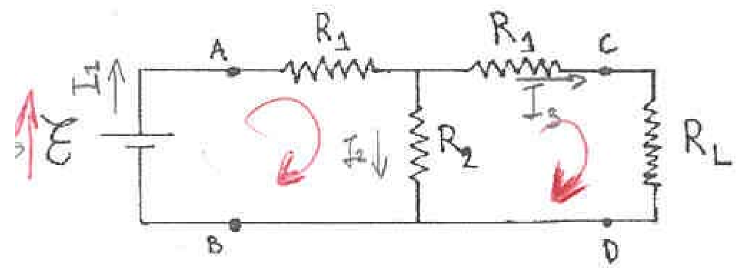
October 8, 2002
Solutions for Practice Quiz #6

The emf source *E* is connected as shown in the figure below in a network that involves resistors R_1 , R_2 and R_L



Let us first introduce the arrows for I1,I2,I3,emf and the "positive" direction in summing potentials. Notice that with the exception of the emf, all other directions are completely arbitrary. Flipping the "positive" direction in summing potentials on a loop simply changes the sign of both hand sides of an equation. For what concerns I1,I2 and I3 though, if we end up with negative values, that means that our initial guess of the direction was wrong and instead the opposite one is the correct.

There are 3 loops that are formed of which 2 equations are independent. There is also a junction equation (charge conservation). There are three unknowns (I1/I2/I3) and we need three (independent) equations in order to determine them. Pick any 2 loop equations and the junction equation, follow Kirchhoff's laws and you are done.

$$\mathcal{E} - I_1 R_1 - I_2 R_2 = 0$$

$$\mathcal{E} - I_1 R_1 - I_3 R_1 - I_3 R_L = 0$$

$$I_1 = I_2 + I_3$$

Notice that there are 3 correct answers to this problem. Which are the other 2?

In calculating R_eff, proceed in steps identifying that you have the right most R_1 in series with R_L. Their sum is in parallel with R_2. This sum is in series with the left most R_1.

$$R_{eff} = R_1 + \frac{(R_1 + R_L) R_2}{R_1 + R_L + R_2}$$

Equate the above R_eff with R_L to find out that:

$$R_{eff} = R_L \Rightarrow R_1^2 + \cancel{R_1 R_L} + R_1 R_2 + R_1 R_2 + \cancel{R_L R_2} = \cancel{R_1 R_L} + R_L^2 + \cancel{R_2 R_L}$$

$$\Rightarrow 2 R_1 R_2 = R_L^2 - R_1^2 \Rightarrow R_2 = \frac{R_L^2 - R_1^2}{2 R_1}$$

In order to find out V_CD, you should identify that I1=emf/R_eff=emf/R_L (remember, R_eff=R_L) and I3=V_CD/R_L. You then have:

$$\mathcal{E} - I_1 R_1 - I_3 (R_1 + R_L) = 0 \Rightarrow \mathcal{E} - \frac{\mathcal{E}}{R_{eff}} R_1 - \frac{V_{CD}}{R_L} (R_1 + R_L) = 0 \Rightarrow$$

$$\mathcal{E} - \frac{\mathcal{E}}{R_L} R_1 - \frac{V_{CD}}{R_L} (R_1 + R_L) = 0 \Rightarrow \mathcal{E} (R_L - R_1) = V_{CD} (R_1 + R_L) \Rightarrow$$

$$V_{CD} = \mathcal{E} \, \frac{R_L - R_1}{R_L + R_1}$$