MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

# Problem Solving 4: Capacitance and Stored Energy

OBJECTIVES

1. To calculate the capacitance of a simple capacitor.

2. To calculate the energy stored in a capacitor in two ways.

REFERENCE: Section 5.2, 8.02 Course Notes.

PROBLEM SOLVING STRATEGIES (see Section 5.9, 8.02 Course Notes)

(1) Using Gauss's Law, calculate the electric field everywhere.

(2) Compute the electric potential difference ΔV between the two conductors.

(3) Calculate the capacitance C using $C = Q / |\Delta V|$.

The Cylindrical Capacitor:
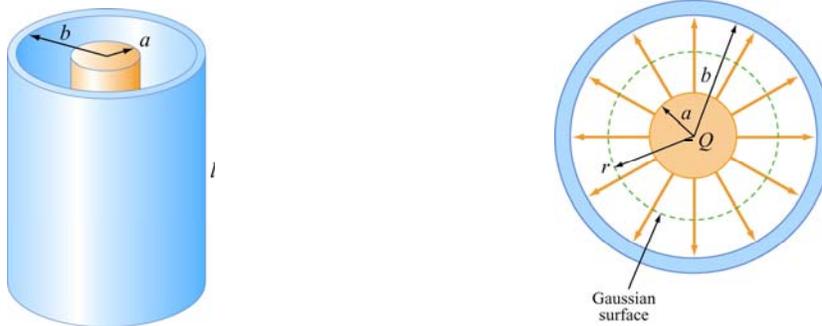
Two concentric cylinders have radii a and b and height l, with $b > a$. The inner cylinder carries total charge $-Q$, and the outer cylinder carries total charge Q.
We ignore end effects.

Note that you have recently calculated the capacitance of a cylindrical capacitor in the pre-lab to experiment #2. You are asked to repeat this calculation here both to ensure that you know how to and to prepare for the newer material of calculating stored energy. Move quickly through the capacitance calculation!

Question 1: The Electric Field

Use Gauss's Law to find the direction and magnitude of the electric field in the between the inner and outer cylinders ($a < r < b$). Express your answer in terms of the total charge Q, the radii a and b, the height l, and any other constants which you may find necessary. NOTE: The inner cylinder has negative charge $-Q$.

Answer (*Write your answer on the tear-sheet at the end!*)

Question 2: Electric Potential Difference (Voltage Difference)

The voltage difference between the cylinders, ΔV, is defined to be the work done per test charge against the electric field in moving a test charge q_t from the inner cylinder to the outer cylinder

$$\Delta V \equiv V(b) - V(a) = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} .$$

Find an expression for the voltage difference between the cylinders in terms of the charge Q, the radii a and b, the height l, and any other constants that you may find necessary.

Answer (*Write your answer on the tear-sheet at the end!*)

Question 3: Calculating Capacitance

Our two conducting cylinders form a capacitor. The magnitude of the charge, $\left|Q\right|$, on either cylinder is related to the magnitude of the voltage difference between the cylinders according to $\left|Q\right| = C\left|\Delta V\right|$ where ΔV is the voltage difference across the capacitor and C is the constant of proportionality called the 'capacitance'. The capacitance is determined by the geometrical properties of the two conductors and is therefore independent of the applied voltage difference across the cylinders.

What is the capacitance C of our system of two cylinders? Express your answer in terms, a, and b, the height l, and any other constants which you may find necessary.

Answer (*Write your answer on the tear-sheet at the end*!)

Question 4: Stored Electrostatic Energy

The total electrostatic energy stored in the electric fields is given by the expression,

$$U = \frac{\varepsilon_0}{2} \int_{all\,space} \vec{\mathbf{E}} \cdot \vec{\mathbf{E}} dV_{vol} \ .$$

Starting from your expression for E in question (1), calculate this electrostatic energy and express your answer in terms of Q, a, b, and l (and any other constants which you may find necessary). If you use your expression for C from question 3 above, can you write you expression in terms of Q and C alone? What is that expression?

**Answer (*Write your answer on the tear-sheet at the end*!)**

Question 5: Charging the Capacitor

Suppose instead of using a battery we charge the capacitor ourselves in the following way. We move charge from the inside of the cylinder at $r = b$ to the surface of the cylinder at $r = a$. Suppose we start off with zero charge on the conductors and we move charge for awhile until at time t we have built up a change $q(t)$ on the inner cylinder.

a. What is the voltage difference between the two cylinders at time t, in terms of C and $q(t)$?

b. Now we move a very little additional charge dq from the outer to the inner cylinder. How much work dW do we have to do to move that dq from the outer to the inner cylinder, in the presence of the charge $q(t)$ already there, in terms of C, $q(t)$, and dq?

c. Using your result in (b), calculate the total work we have to do to bring a total charge Q from the outer to the inner cylinder, assuming the cylinders start out uncharged (Hint: integrate with respect to dq from 0 to Q).

d. Is the work we did in charging the capacitor greater than, equal to, or less than the stored electrostatic energy in the capacitor that you calculated in question 4? Why?

Answer (*Write your answers on the tear-sheet at the end!*)

Sample Exam Question (If time, try to do this by yourself, closed notes)
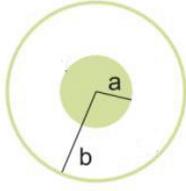
A solid conducting sphere of radius a is surrounded by a thin conducting spherical shell of inner radius b, with $b > a$. The inner sphere carries total charge Q, and the outer sphere carries charge $-Q$.

(a) Using Gauss's Law, find an expression for electric field $\vec{E}$ at points $a < r < b$. Show the Gaussian surface you use.

(b) Using your expression for $\vec{E}$, find the electric potential difference between the inner and outer conductor, that is $\Delta V \equiv V(a) - V(b) = -\int_b^a \vec{E} \cdot d\vec{s}$

(c) Using you results from (b), derive an expression for the capacitance of this capacitor, in terms of the quantities given.

(d) Calculate the total electrostatic energy U by using your expression from (a) and integrating the energy density $\frac{1}{2}\varepsilon_o E^2$ over the volume. The proper volume element here is $dV_{volume} = 4\pi r^2 dr$

(e) Show that your answer in (d) agrees with the formula $\frac{1}{2}CV^2$ or $\frac{1}{2}\frac{Q^2}{C}$, which ever expression you prefer, using the capacitance C from (c).

Solving4-5

Problem Solving 4: Calculating Capacitance and Stored Energy

Group _____ **(e.g. L02 10A Please Fill Out)**

Names _____

_____

_____

Question 1: The Electric Field

Use Gauss's Law to find the direction and magnitude of the electric field in the between the inner and outer cylinders ($a < r < b$). Express your answer in terms of the total charge Q, the radii a and b, the height l, and any other constants which you may find necessary.

Answer:

Question 2: Electric Potential Difference (Voltage Difference):

Find an expression for the voltage difference between the cylinders in terms of the charge Q, a, b, and l (and any other constants which you may find necessary).

Answer:

Question 3: Calculating Capacitance

What is the capacitance C of our system of two cylinders? Express your answer in terms of a, and b, the height l, and any other constants which you may find necessary.

Answer:

Question 4: Stored Electrostatic Energy

What is the total electrostatic energy?   If you use your expression for C from question 3 above, can you write you expression in terms of Q and C alone? What is that expression?

Answer:

Question 5:   Charging the Capacitor

a.   What is the voltage difference between the two cylinders at time t, in terms of C and $q(t)$?

b.   How much work dW do we have to do to move that dq from the outer to the inner cylinder, in the presence of the charge $q(t)$ already there, in terms of C, $q(t)$, and dq?

c.   Using your result in (b), calculate the total work we have to do to bring a total charge Q from the outer to the inner cylinder, assuming the cylinders start out uncharged (Hint: integrate with respect to dq from 0 to Q).

d.   Is the work we did in charging the capacitor greater than, equal to, or less than the stored electrostatic energy in the capacitor that you calculated in question 4? Why?