Summary of Class 04 **8.02**

Topics: Working in Groups, Electric Potential, E from V
Related Reading:
Course Notes (Liao et al.): Sections 3.1-3.5

Topic Introduction

We first discuss groups and what we expect from you in group work. We then turn to the concept of electric potential. Just as electric fields are analogous to gravitational fields, electric potential is analogous to gravitational potential. We introduce from the point of view of calculating the electric potential given the electric field. Next we consider the opposite process, that is, how to calculate the electric field if we are given the electric potential.

Potential Energy

Before defining potential, we first remind you of the more intuitive idea of potential energy. You are familiar with gravitational potential energy, U (= mgh in a uniform gravitational field g, such as is found near the surface of the Earth), which changes for a mass m only as that mass changes its position. To change the potential energy of an object by ΔU, one must do an equal amount of work W_{ext}, by pushing with a force $\boldsymbol{F_{ext}}$ large enough to move it:

$$\Delta U = U_B - U_A = \int_A^B \vec{\mathbf{F}}_{\mathbf{ext}} \cdot d\,\vec{\mathbf{s}} = W_{ext}$$

How large a force must be applied? It must be equal and opposite to the force the object feels due to the field it is sitting in. For example, if a gravitational field g is pushing down on a mass m and you want to lift it, you must apply a force mg upwards, equal and opposite the gravitational force. Why equal? If you don't push enough then gravity will win and push it down and if you push too much then you will accelerate the object, giving it a velocity and hence kinetic energy, which we don't want to think about right now.
This discussion is generic, applying to both gravitational fields and potentials and to electric fields and potentials. In both cases we write:

$$\Delta U = U_B - U_A = -\int_A^B \vec{\mathbf{F}} \cdot d\,\vec{\mathbf{s}}$$

where the force $\boldsymbol{F}$ is the force the field exerts on the object.
Finally, note that we have only defined *differences* in potential energy. This is because only differences are physically meaningful – what we choose, for example, to call "zero energy" is completely arbitrary.

Potential

Just as we define electric fields, which are created by charges, and which then exert forces on other charges, we can also break potential energy into two parts: (1) charges create an electric potential around them, (2) other charges that exist in this potential will have an associated potential energy. The creation of an electric potential is intimately related to the creation of an electric field: $\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$. As with potential energy, we only define a potential difference. We will occasionally ask you to calculate "the potential," but in these cases we must arbitrarily assign some point in space to have some fixed potential. A common assignment is to call the potential at infinity (far away from any charges) zero. In order to find the potential anywhere else you must integrate from this place where it is known (e.g. from $A=\infty$, $V_A=0$) to the place where you want to know it.

Once you know the potential, you can ask what happens to a charge q in that potential. It will have a potential energy $U = qV$. Furthermore, because objects like to move from high potential energy to low potential energy, as long as the potential is not constant, the object will feel a force, in a direction such that its potential energy is reduced. Mathematically that is the same as saying that $\vec{\mathbf{F}} = -\nabla U$ (where the gradient operator $\nabla \equiv \frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}$) and hence, since $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$, $\vec{\mathbf{E}} = -\nabla V$. That is, if you think of the potential as a landscape of hills and valleys (where hills are created by positive charges and valleys by negative charges), the electric field will everywhere point the fastest way downhill.

Configuration Energy
Since moving a charge through a potential difference takes energy (it changes the potential energy of the charge), we can also discuss the total amount of energy that it would take to assemble a collection of charges, assuming that they started a very far distance apart ("at infinity") and then were brought in to their final positions. A straight-forward way to think about, and calculate, this is to bring the charges in one at a time. The first one is "free" – it doesn't see a potential. The second charge is brought in through the potential created by the first. The third sees the potential from the first two, and so forth.

Important Equations

Potential Energy (Joules) Difference: $\Delta U = U_B - U_A = -\int_A^B \vec{\mathbf{F}} \cdot d\,\vec{\mathbf{s}}$

Electric Potential Difference (Joules/Coulomb = Volt): $\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$

Electric Potential (Joules/coulomb) created by point charge: $V_{\text{Point Charge}}(r) = \frac{kQ}{r}$

Potential energy U (Joules) of point charge q in electric potential V: $U = qV$

Configuration Energy: $U = \sum_{all\ pairs} \frac{q_i q_j}{4\pi\varepsilon_o \left|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j\right|}$