

Let's consider a ball that is dropped from a certain height,  $h_i$ , above the ground and this ball is falling.

It hits the ground and it bounces up until it reaches some final height,  $h_{final}$ .

Now when the ball is colliding with the ground, there are collision forces.

And in this problem what we like to do is figure out what the average force of the ground is on the ball.

And that will be the normal force, the average normal force, on the ball during the collision.

Now if we look at this ball dropping, it's going to lose a little bit of energy, because it's getting compressed at the collision.

Let's look at an example of the actual ball dropping.

As you can see in this high speed video, as the ball falls down, it collides with the ground.

When it collides with the ground, it's compressed.

And then as it rebounds upwards, the ball expands back to its original shape, but it doesn't quite get to the same height-- that's because when the ball is compressed, there's some deformation in the rubber structure of the ball, and it's not a completely elastic deformation.

And so some of the energy is transformed into, first, molecular motions, which turn into thermal energy that's radiative into the environment.

Let's look in particular at the details of the collision.

If we look at it in slow motion what we have here-- and I'll draw a picture-- as the ball is colliding with the ground, ball compresses, expands as it goes upwards.

And so we can draw a free body diagram of the ball with a normal force and a gravitational force.

Now let's choose our positive direction up.

So now what we'd like to do is apply the momentum principle to analyze the average normal force.

And our momentum principle, remember is impulse.

The force integrated over some time during the collision is equal to the change in momentum.

So what we'd like to do is identify the states that are relevant.

So it will have a state before, so what we'll do is we'll call this the before state, and that's right before the ball is hitting the ground.

And we have an after state, and in the after state, the ball has now finished colliding with the ground, and it's now moving up with speed up.

Now again, we're going to choose a positive up.

Here on representing things as it speeds.

One of the things, we need some times here, so let's say that  $t_{\text{initial}}$  is zero, this is our final time.

We'll call this time the before time, we'll just call this  $t_{\text{before}}$ , and this is  $t_{\text{after}}$ .

And then our integral is going from before to after the momentum.

And we can now apply the momentum principle.

Well, this is a vector equation and we've chosen unit vectors up, so what we have here is the integral of from  $t_{\text{before}}$  to  $t_{\text{after}}$  of  $N \text{ minus } mg$ , integrated over  $dt$ , and that's equal to the momentum at the  $y$  component of the momentum at  $t_{\text{after}}$ , minus the  $y$  component of the momentum.

We don't have a vector here anymore.

The  $y$  component of the vector,  $t_{\text{before}}$ .

And so this is our expression of the momentum principle.

Impulse causes momentum to change.

Now we're assuming that the normal force just averaging it and so this integral simply becomes  $N_{\text{average}} \text{ minus } mg$ , times the time of collision, is equal to-- now in here we can put the mass of the ball, we have the velocity.

Now here's where we have to be a little bit careful, because we're looking at the  $y$  component.

We chose speed downwards, that's in the negative  $y$  direction, so we have minus-- sorry, we're looking at after.

We have plus  $V_{\text{after}}$ , because this is going in the positive  $j$  direction.

And over here we have a negative mass, but it's going in the minus direction, so we have a minus  $mV_{\text{before}}$ , and

so we get mass times  $V$  after plus  $V$  before.

So our first result is that the normal force average.

Let's bring the divide through by  $\Delta t$ , and bring the  $Ng$  term over, so we have  $m V_a$  plus  $V_b$ , divided by  $\Delta t$ , plus  $mg$ .

So we see that if the collision time is very short, then this average force is a little bit bigger.

A long collision time, the average force a little bit smaller.

Now from kinematics, we already have worked out the problem that the speed for an object that rises to a height,  $h$  final, this is the velocity afterwards, is just square root of  $2g h$  final.

And in a similar way, if an object is falling height  $h$  i, the speed when it gets to the bottom is  $2gh_i$ .

And so now we can conclude with these substitutions that the average force equals  $m$  times square root of  $2g h$  final, plus the square root of  $2g h$  initial, over the collision time, plus and  $mg$ .

And of course the collision time, we're saying is  $t$  after minus  $t$  before.

And so that's how we can use the momentum principle to get an average expression for the normal force.