5.1 Introduction to the Vector Description of Motion in Two Dimensions

We have introduced the concepts of position, velocity and acceleration to describe motion in one dimension; however we live in a multidimensional universe. In order to explore and describe motion in more than one dimension, we shall study the motion of a projectile in two-dimension moving under the action of uniform gravitation.

We extend our definitions of position, velocity, and acceleration for an object that moves in two dimensions (in a plane) by treating each direction independently, which we can do with vector quantities by resolving each of these quantities into components. For example, our definition of velocity as the derivative of position holds for each component separately. In Cartesian coordinates, the position vector $\vec{\mathbf{r}}(t)$ with respect to some choice of origin for the object at time t is given by

$$\vec{\mathbf{r}}(t) = x(t)\,\hat{\mathbf{i}} + y(t)\,\hat{\mathbf{j}}. \tag{5.1.1}$$

The velocity vector $\vec{\mathbf{v}}(t)$ at time t is the derivative of the position vector,

$$\vec{\mathbf{v}}(t) = \frac{dx(t)}{dt}\,\hat{\mathbf{i}} + \frac{dy(t)}{dt}\,\hat{\mathbf{j}} \equiv v_x(t)\,\hat{\mathbf{i}} + v_y(t)\,\hat{\mathbf{j}}, \tag{5.1.2}$$

where $v_x(t) \equiv dx(t)/dt$ and $v_y(t) \equiv dy(t)/dt$ denote the x- and y-components of the velocity respectively.

The acceleration vector $\vec{\mathbf{a}}(t)$ is defined in a similar fashion as the derivative of the velocity vector,

$$\vec{\mathbf{a}}(t) = \frac{dv_x(t)}{dt}\,\hat{\mathbf{i}} + \frac{dv_y(t)}{dt}\,\hat{\mathbf{j}} \equiv a_x(t)\,\hat{\mathbf{i}} + a_y(t)\,\hat{\mathbf{j}}, \tag{5.1.3}$$

where $a_x(t) \equiv dv_x(t)/dt$ and $a_y(t) \equiv dv_y(t)/dt$ denote the x- and y-components of the acceleration.