4.4 Acceleration

We shall apply the same physical and mathematical procedure for defining acceleration, as the rate of change of velocity with respect to time. We first consider how the instantaneous velocity changes over a fixed time interval of time and then take the limit as the time interval approaches zero.

4.4.1 Average Acceleration

Average acceleration is the quantity that measures a change in velocity over a particular time interval. Suppose during a time interval Δt a body undergoes a change in velocity

$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}(t+\Delta t) - \vec{\mathbf{v}}(t)\,. \qquad (4.3.22)$$

The change in the x-component of the velocity, Δv, for the time interval $[t, t+\Delta t]$ is then

$$\Delta v = v(t+\Delta t) - v(t)\,. \qquad (4.3.23)$$

The x***-component of the average acceleration*** for the time interval Δt is defined to be

$$\vec{\mathbf{a}}_{ave} = a_{ave}\,\hat{\mathbf{i}} \equiv \frac{\Delta v}{\Delta t}\,\hat{\mathbf{i}} = \frac{(v(t+\Delta t) - v(t))}{\Delta t}\,\hat{\mathbf{i}}\,. \qquad (4.3.24)$$

The SI units for average acceleration are meters per second squared, $[\mathrm{m\cdot s^{-2}}]$.

4.4.2 Instantaneous Acceleration

Consider the graph of the x-component of velocity, $v(t)$, (Figure 4.7).

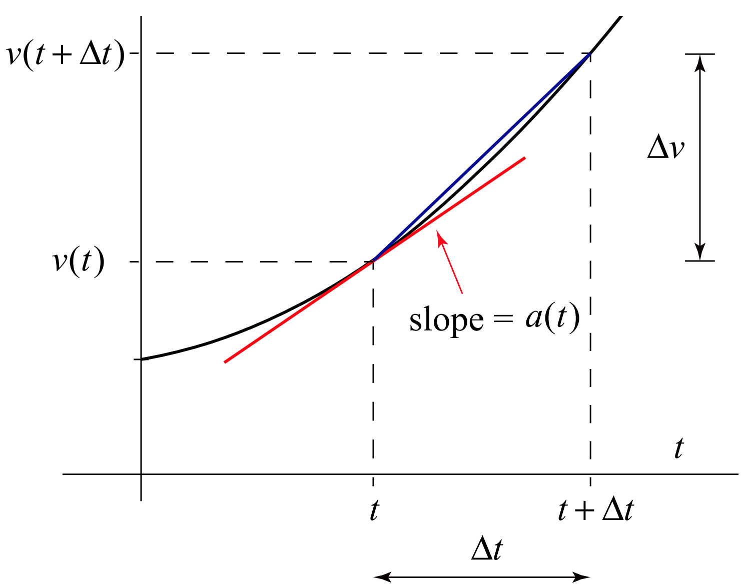

Figure 4.7 Graph of velocity *vs*. time showing the tangent line at time t.

The average acceleration for a fixed time interval Δt is the slope of the straight line connecting the two points $(t, v(t))$ and $(t+\Delta t, v(t+\Delta t))$. In order to define the x-component of the instantaneous acceleration at time t, we employ the same limiting argument as we did when we defined the instantaneous velocity in terms of the slope of the tangent line.

> The x***-component of the instantaneous acceleration*** *at time* t is the slope of the tangent line at time t of the graph of the x-component of the velocity as a function of time,

$$a(t) \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{(v(t+\Delta t) - v(t))}{\Delta t} \equiv \frac{dv}{dt}. \tag{4.3.25}$$

The instantaneous acceleration vector at time t is then

$$\vec{\mathbf{a}}(t) = a(t)\,\hat{\mathbf{i}}. \tag{4.3.26}$$

Because the velocity is the derivative of position with respect to time, the x-component of the acceleration is the second derivative of the position function,

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}. \tag{4.3.27}$$

Example 4.3 Determining Acceleration from Velocity

Let's continue Example 4.1, in which the position function for the body is given by $x = x_0 + (1/2)bt^2$, and the x-component of the velocity is $v = bt$. The x-component of the instantaneous acceleration is the first derivative (with respect to time) of the x-component of the velocity:

$$a = \frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{v(t+\Delta t) - v(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{bt + b\Delta t - bt}{\Delta t} = b. \tag{4.3.28}$$

Note that in Eq. (4.3.28), the ratio $\Delta v / \Delta t$ is independent of t, consistent with the constant slope as shown in Figure 4.5.