- Last Lecture
  - Energy and Momentum of rotation
- Today
  - More about Momentum of rotation
- Important Concepts
  - Equations for angular motion are mostly identical to those for linear motion with the names of the variables changed.
  - Kinetic energy of rotation adds a new term to the same energy equation, it does not add a new equation.
  - Momentum of rotation gives an additional equation
    - There is the additional complication that the moment of inertia can change.

Moment of Inertia

- $I = \Sigma m_i r_i^2 = \int r^2 dm$
  - Hoop (all mass at same radius) $I=MR^2$
  - Solid cylinder or disk $I=(1/2)MR^2$
  - Rod around end $I=(1/3)ML^2$
  - Rod around center $I=(1/12)ML^2$
  - Solid sphere $I=(2/5)MR^2$
- The same object could have a different moment of inertia depending on the choice of axis.
- In the equation: $\Sigma\vec{\tau} = I\vec{\alpha}$ all three quantities need to be calculated using the same axis.

Angular Momentum

- Conserved when external torques are zero or when you look over a very short period of time.
  - True for any fixed axis and for the center of mass
- Formula we will use is simple: $\vec{L} = I\vec{\omega}$
  - Vector nature (CW or CCW) is still important
- Point particle: $\vec{L} = \vec{r} \times \vec{p}$
- Conservation of angular momentum is a separate equation from conservation of linear momentum
- Angular impulse: $\vec{\tau} = \frac{d\vec{L}}{dt} \qquad \Delta\vec{L} = \int \vec{\tau} dt$

Torque Checklist

- Make a careful drawing showing ***where*** forces act
  - Clearly indicate what axis you are using
  - Clearly indicate whether CW or CCW is positive
- For each force:
  - If force acts at axis or points to or away from axis, $\tau=0$
  - Draw (imaginary) line from axis to point force acts. If distance and angle are clear from the geometry $\tau=Fr\sin(\theta)$
  - Draw (imaginary) line parallel to the force. If distance from axis measured perpendicular to this line (lever arm) is clear, then the torque is the force times this distance
- Don't forget CW versus CCW, is the torque + or –