- Last Lecture
  - Pendulums and Kinetic Energy of rotation
- Today
  - Energy and Momentum of rotation
- Important Concepts
  - Equations for angular motion are mostly identical to those for linear motion with the names of the variables changed.
  - Kinetic energy of rotation adds a new term to the same energy equation, it does not add a new equation.
  - Momentum of rotation gives an additional equation

Important Reminders

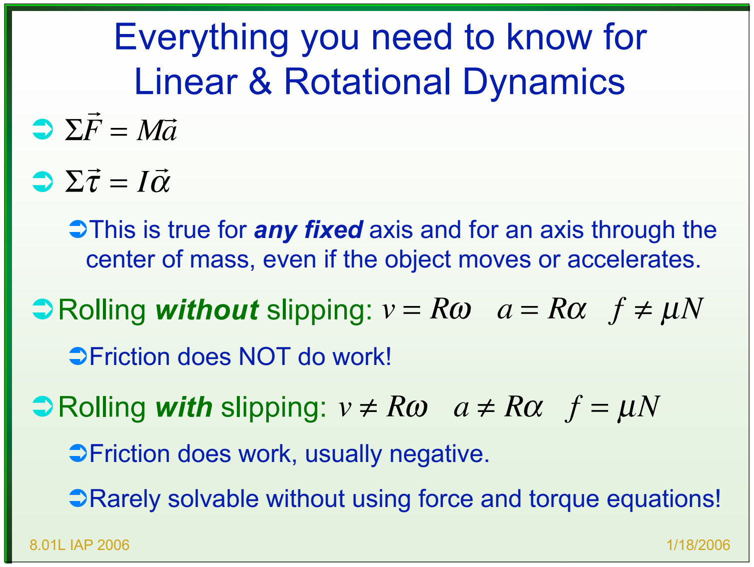
- Contact your tutor about session scheduling
- Mastering Physics due today at 10pm.
- Pset due this Friday at 11am.

Torque Checklist

- Make a careful drawing showing ***where*** forces act
  - Clearly indicate what axis you are using
  - Clearly indicate whether CW or CCW is positive
- For each force:
  - If force acts at axis or points to or away from axis, $\tau=0$
  - Draw (imaginary) line from axis to point force acts. If distance and angle are clear from the geometry τ=Frsin(θ)
  - Draw (imaginary) line parallel to the force. If distance from axis measured perpendicular to this line (lever arm) is clear, then the torque is the force times this distance
- Don't forget CW versus CCW, is the torque + or –

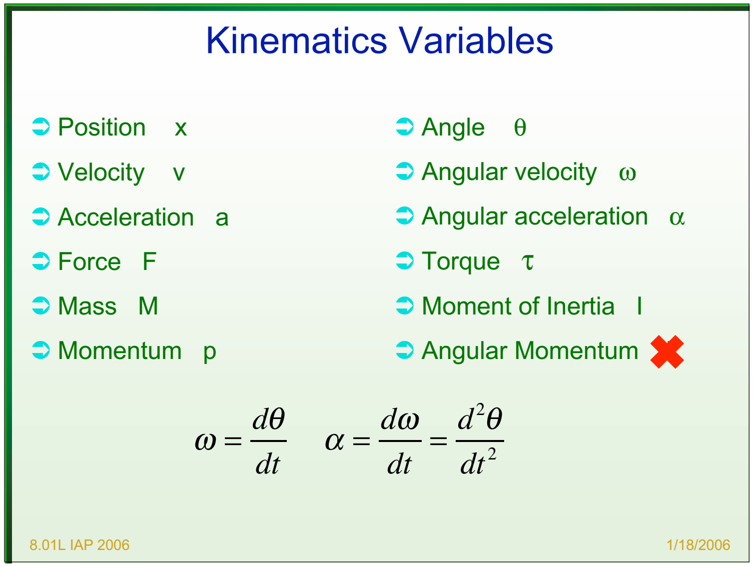
Kinetic Energy with Rotation

- Adds a new term not a new equation!
- Rotation around any fixed pivot: $KE = \frac{1}{2} I_{pivot}\omega^2$
- Moving and rotating: $KE = \frac{1}{2} I_{CM}\omega^2 + \frac{1}{2} M_{Tot} v_{CM}^2$



Everything you need to know for Linear & Rotational Dynamics

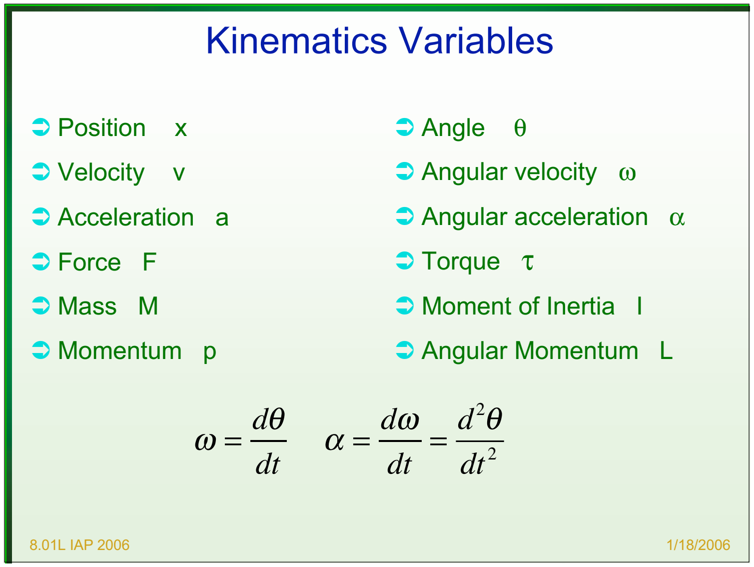
- $\Sigma\vec{F} = M\vec{a}$
- $\Sigma\vec{\tau} = I\vec{\alpha}$
  - This is true for ***any fixed*** axis and for an axis through the center of mass, even if the object moves or accelerates.
- Rolling ***without*** slipping: $v = R\omega \quad a = R\alpha \quad f \neq \mu N$
  - Friction does NOT do work!
- Rolling ***with*** slipping: $v \neq R\omega \quad a \neq R\alpha \quad f = \mu N$
  - Friction does work, usually negative.
  - Rarely solvable without using force and torque equations!



Kinematics Variables

Position	x	Angle	θ
Velocity	v	Angular velocity	ω
Acceleration	a	Angular acceleration	α
Force	F	Torque	τ
Mass	M	Moment of Inertia	I
Momentum	p	Angular Momentum	✖

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$



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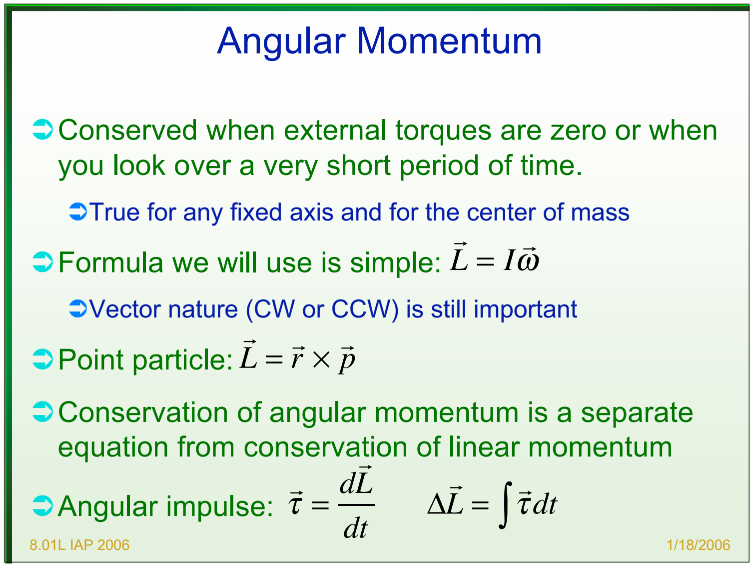

Angular Momentum

- Conserved when external torques are zero or when you look over a very short period of time.
  - True for any fixed axis and for the center of mass
- Formula we will use is simple: $\vec{L} = I\vec{\omega}$
  - Vector nature (CW or CCW) is still important
- Point particle: $\vec{L} = \vec{r} \times \vec{p}$
- Conservation of angular momentum is a separate equation from conservation of linear momentum
- Angular impulse: $\vec{\tau} = \frac{d\vec{L}}{dt} \qquad \Delta\vec{L} = \int \vec{\tau}\,dt$