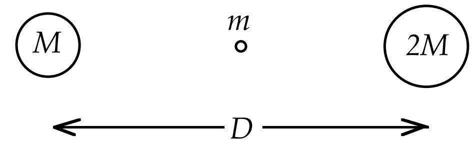
8.01L **Fall 2005**

Exam 3 Sample Problems

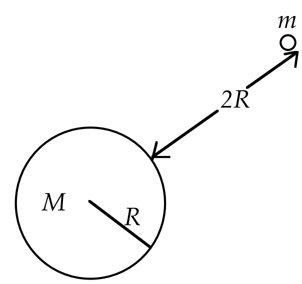
Note that some of these problems were from an exam allowing calculators. Your exam will not allow calculators.

Problem 1

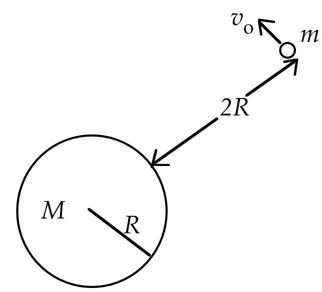
a) Two large spherical stars with masses of M and $2M$ are positioned a distance D apart (measured from the center of one star to the center of the other star) as shown. A small spherical asteroid with mass m is located with its center exactly halfway between the two large stars. Find the magnitude and direction of the total gravitational force acting on the asteroid.



b) A small probe of mass m is released from rest at a distance of $2R$ above the surface of a spherical planet of mass M and radius R. Find the speed that the probe will have when it hits the surface of the planet. Assume that the planet does not have an atmosphere and no forces other than gravity act on the probe.



c) Assume that the probe is released at the same distance from the planet but now with a velocity of $v_o = \sqrt{GM/4R}$. The probe has a rocket which fires to put it into a circular orbit at that height. Find the speed the probe would have once it is in the circular orbit and, using that, find the work done by the rocket. Assume that, at all times, the probe is at the same distance from the planet and that there are no forces acting on the probe other than the planet's gravity and the rocket.

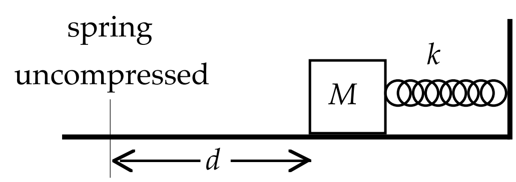


Extra Credit Question This question may require a **lot** of thought for the points available. Try it **only** if you have finished with all other problems

d) Redo part (b) but assuming that the planet is a hollow shell and that the probe falls through a hole in the shell. Find the speed of the probe when it gets to the center of the hollow shell. Explain your answer.

Problem 2

a) An object of mass, M=2 kg, is attached to a spring of spring constant k=50 N/m which is *compressed* a distance d=20 cm and then released at rest. Find the speed of the object when it has gone past the point where the spring is uncompressed and now the spring is *stretched* a distance of 10 cm. Assume that the mass is moving on a horizontal, frictionless surface.



b) Write an equation for the position of the mass as a function of time with t=0 being the instant that the mass was first released from rest. Use this equation to find out how long it takes the mass to get from the initial point with the spring *compressed* by 20 cm to the point where the spring is *stretched* by 10 cm. (Hint: Think carefully about the units of ω when doing the trig functions on your calculator.)

c) Write an equation for the velocity as a function of time, using the same definition of t=0 as in part (b). Use this equation to find the velocity (magnitude and direction) when the time is $t = 3T/4$ where T is the period of motion of the mass on the spring.

Problem 3

Two hockey pucks collide on a horizontal, frictionless surface. The velocities of both pucks before the collision are shown in the drawing. After they collide, puck A is moving as shown. Assume that the mass of puck A is twice as large as the mass of puck B: $M_A = 2*M_B$.

a) Find the magnitude and direction of the velocity of puck B after the collision. Clearly indicate on your drawing the angle you are using to specify the direction of puck B.
b) Is kinetic energy conserved in this collision? Justify your answer. If not, clearly indicate whether KE is gained or lost.

Problem 4

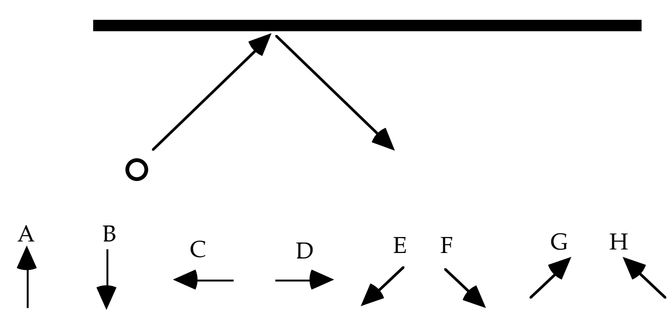
a) A baseball-player-turned-astronaut stands on Mimas, a small moon of Saturn, and throws a baseball with initial speed $37 m/s$. The mass of the baseball is $0.15 kg$; the mass of Mimas is $3.8 \text{x} 10^{19} kg$ and its radius is $200 km$. If the astronaut throws the ball horizontally, will it go into a circular orbit around Mimas? Explain your answer.
b) For the same situation given in part b), if the astronaut throws the ball vertically, how high above the surface of Mimas will it rise before stopping?

Problem 5 Short Answer Questions:

You must show your work or write an explanation of your answer to get credit for these problems.

i) A ball hits a wall and bounces off as shown. Assume that the collision is elastic. Which vector best represents the direction of the change in momentum of the ball?

a) Circle your choice and explain it b) Not enough information to answer.



ii) A small car is moving along a straight level highway at a speed of 3v. The car hits from behind a large truck moving in the same direction at speed v. After the collision, the car is stuck onto the truck. During the collision, which vehicle experiences the greater average force? Explain your answer.

a) The car b) The truck c) The forces are equal

d) Impossible to determine without information about the masses

iii) Two balls of masses 2 kg and 3 kg slide along a frictionless horizontal surface with speeds of 4 m/s and 2 m/s, respectively. After an inelastic collision, the balls stick together and move at a speed of 2 m/s. What direction did the two balls move before the collision?

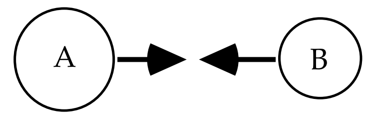
a) In the same direction b) In opposite directions

c) At an angle not equal to 0° or 180° d) The situation described is impossible

e) Not enough information given to select an answer

Problem 6

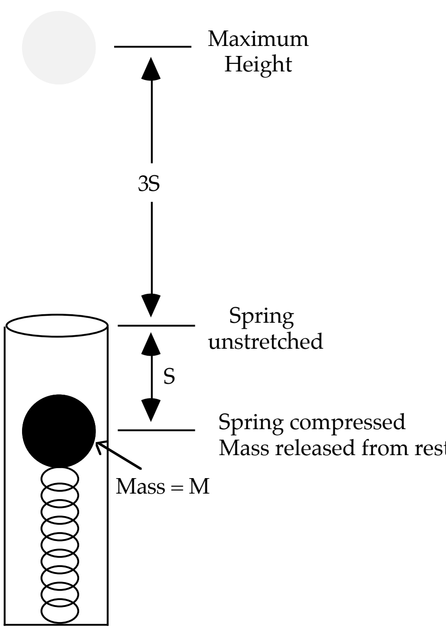
Two steel spheres are shot at each other and then collide head-on as shown. Sphere A has a mass of 10 kg and a velocity just before the collision of 3 m/s to the right. Sphere B has mass of 4 kg and velocity just before the collision of 4 m/s to the left. Immediately after the collision the velocity of sphere B is observed to be 6 m/s to the right.



a) What is the magnitude and direction of the velocity of sphere A immediately after the collision? Clearly indicate your coordinate system and what direction is positive.
b) Is this collision elastic or inelastic? Explain your answer.
c) Assume that the collision lasts 10^{-3} seconds. Calculate the magnitude and direction of the average force that sphere A exerts on sphere B during the collision. Clearly indicate your coordinate system and what direction is positive.

Problem 7

A spring-loaded toy gun is used to shoot a ball of mass M straight up in the air. The ball is not attached to the spring. The ball is pushed down onto the spring so that the spring is compressed a distance S below its unstretched point. After release, the ball reaches a maximum height 3S, measured from the unstretched position of the spring (see diagram).



a) Find the spring constant of the spring.

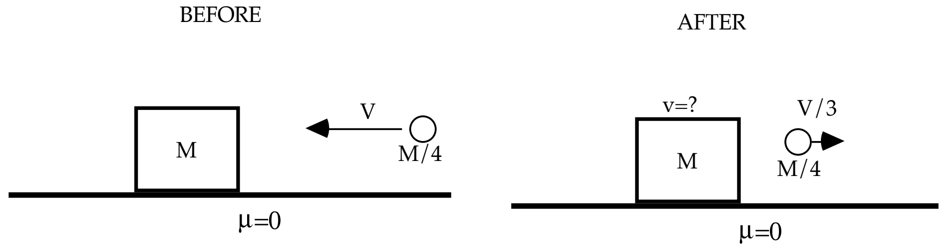
b) Find the equilibrium point of the ball when it is sitting on the spring with no forces other than gravity and the spring acting on it. Clearly indicate the point you are using as the origin of your coordinate system and what direction is positive.
c) Now, the ball is glued onto the spring so that it oscillates up and down rather than flying off the spring. The spring is again compressed the same distance S below its **unstretched** point. Write an equation for the position of the ball as a function of time after it is released. Clearly indicate the point you are using as the origin of your coordinate system and what direction is positive.

Problem 8

A projectile is launched straight up. It explodes into two pieces at the top point of its trajectory. One piece has twice the mass of the other one. The more massive piece has kinetic energy equal to K right after the explosion. Show that the total energy released in the explosion (i.e. the total kinetic energy of the two pieces right after the explosion) is exactly equal to 3.0K. Justify your answer with a calculation.

Problem 9

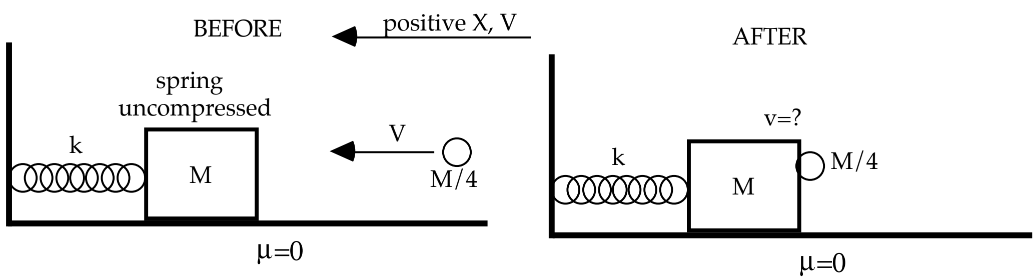
A metal block of mass M is free to slide on a frictionless, horizontal surface. A metal ball of mass M/4 is fired at the block with velocity V, and bounces straight backward off the block with one-third its original speed. The block is initially at rest.



a) What is the speed of the block after the impact?
b) Is this collision elastic?
c) If the impact lasts Δt sec, what average force (magnitude and direction) acts on the block?
d) How does the average force (magnitude and direction) that acts on the ball compare to what you found in part (c) for the force on the block. Explain your answer.

Problem 10

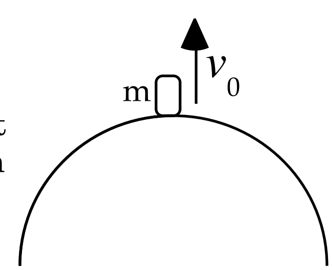
A metal block of mass M is attached to a spring of negligible mass and spring constant k as shown, and is free to slide on a frictionless, horizontal surface. A clay ball of mass M/4 is fired at the block with velocity V, and sticks to it as shown. The block is initially at rest and the spring is initially uncompressed.



a) What is the speed of the block+ball immediately after the impact?
b Write an equation for the position of the block as a function of time after the collision, assuming that at $t = 0$, the instant of the impact, it is at $x = 0$ which is the unstretched point of the spring. Determine values for the amplitude, angular frequency, and phase in terms of the given quantities. Assume that X and V are positive to the left as shown.

Problem 11

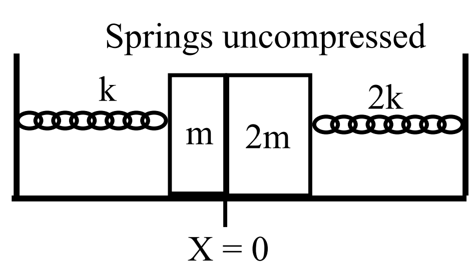
A small object of mass m is launched from the surface of the Earth with a speed of v_0 in a direction perpendicular to the Earth's surface.



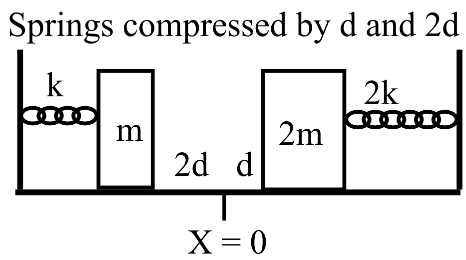
a) What is the total mechanical energy of the object at its starting point in terms of m, v_0, the radius of the Earth R_e, the mass of the Earth M_e, and the gravitational constant G ?
b) Find an expression for the speed v of the object at a height $h = R_e$ (i.e., a distance $2R_e$ from Earth's center).
c) What is the minimum value of v_0 that will allow the object to reach the height h?
d) Now consider a different situation where the object is placed in a circular orbit at a height $h = R_e$ (i.e., a distance $2R_e$ from Earth's center). Find the velocity the object needs to be in a circular orbit at that height.

Problem 12 Short Answer Questions

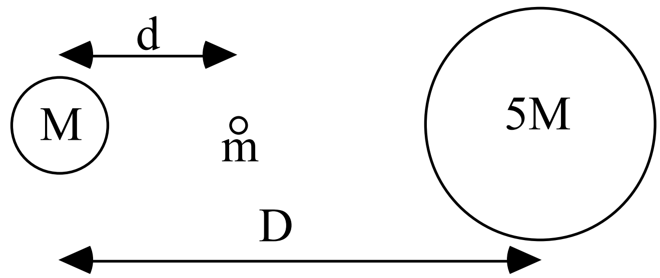
A) Two blocks move on a horizontal, frictionless surface and are attached to springs as shown. The left block has mass **m** and spring constant **k**. The right block has mass **2m** and spring constant **2k**. The masses just touch each other at X=0 at which point both springs are at equilibrium (i.e. uncompressed). The left mass is pulled to the left a distance



2d and the right mass is pulled to the right a distance **d**. Both masses are released from rest at the same time. Explain why both masses get back to X=0 at the same time.



B) The diagram shows a binary star system with one star of mass **M** and the other of mass **5M** with the centers of the two stars separated by a distance **D** as shown. Assume that a third small object of mass **m** is located a distance **d** from the center of the left star. The centers of all three objects are on the same line.



i) Is there a value of d for which the total gravitational force on the small mass is zero? If so, write the equation you would solve to find this value of d. Note that you do **NOT** need to find the value of d. If there is no such value, explain why not.

ii) Using the standard equation for potential energy due to gravity far from the surface, is there a value of d for which the total gravitational potential energy of the small mass is zero? If so, write the equation you would solve to find this value of d. Note that you do **NOT** need to find the value of d. If there is no such value, explain why not.

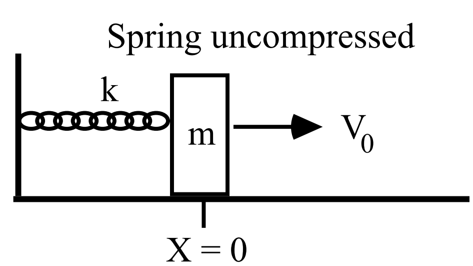
Problem 13

Rocket A propels an object of mass **m** from the surface of the Earth to an altitude $\mathbf{3R_E}$ ($\mathbf{R_E}$ is the radius of the Earth) above the surface of the Earth at which point the object stops momentarily before falling back. Rocket B propels an object of the same mass into a circular orbit at an altitude of $\mathbf{R_E}$ above the surface. Which rocket does more work? Assume that the initial velocity due to the rotation of the Earth can be ignored. Justify your answer.

Problem 14

An object of mass **m** is attached to a spring of spring constant **k** and moves on a frictionless horizontal surface. The mass is at X=0 (where the spring is unstretched) and it has an initial velocity of $\mathbf{V_0}$ in the positive X direction as shown. The subsequent position and velocity of the object as a function of time are given by:

$$X(t) = A\sin(Ct) \quad \text{and} \quad V(t) = B\cos(Ct)$$



a) Find the values of A, B, and C in terms of **k**, **m**, and $\mathbf{V_0}$.

b) Since X=0 at t=0, the average velocity between t=0 and a later time can be written simply as

$$V_{avg}(t)=X(t)/t$$

Find the average velocity between t=0 and t=T/2, where T is the period of oscillation of the mass on the spring. Hint: If you think carefully, you can get the answer with almost no calculations.

c) Find the average velocity between t=0 and t=T/3, where T is the period of oscillation of the mass on the spring. Express your answer in terms of $\mathbf{V_0}$ only. Your answer can contain trigonometric functions (like sin, cos, tan, etc.) and numerical constants but no variables other than $\mathbf{V_0}$.