Massachusetts Institute of Technology
Department of Physics

Physics 8.01L

SAMPLE EXAM 3 SOLUTIONS December 4, 2005

## Problem 1

**a)** $F_{left} = \frac{GMm}{(D/2)^2} = \frac{4GMm}{D^2}, \quad F_{right} = \frac{G(2M)m}{(D/2)^2} = \frac{8GMm}{D^2}$
$F_{TOT} = F_{right} - F_{left} = \frac{8GMm}{D^2} - \frac{4GMm}{D^2}$
$F_{TOT} = \frac{4GMm}{D^2}$, to the right.

**b)** Mechanical energy is conserved.
$PE_I + KE_I = PE_F + KE_F \;\Rightarrow\; \frac{-GMm}{3R} + 0 = \frac{-GMm}{R} + \frac{1}{2}mv^2$
$\frac{1}{2}v^2 = \frac{-GM}{3R} + \frac{GM}{R} = \frac{2GM}{3R} \Rightarrow v^2 = \frac{4GM}{3R}$
$v = \sqrt{\frac{4GM}{3R}}$

**c)** For a circular orbit at distance $3R$: $m\frac{v^2}{3R} = \frac{GMm}{(3R)^2} \Rightarrow v = \sqrt{\frac{GM}{3R}}$
$W_{rocket} = \Delta E, \quad \Delta PE = 0$, because always at the same distance.
$W_{rocket} = KE_F - KE_I = \frac{1}{2}mv_F^2 - \frac{1}{2}mv_I^2 = \frac{1}{2}m\frac{GM}{3R} - \frac{1}{2}m\frac{GM}{4R} = \frac{GMm}{R}\left(\frac{1}{6} - \frac{1}{8}\right)$
$W_{rocket} = +\frac{GMm}{24R}$

**d)** Force $= 0$ inside shell $\Rightarrow PE = constant \Rightarrow KE = constant$.
$\Rightarrow$ Answer is the same as for part **(b)**, $v = \sqrt{\frac{4GM}{3R}}$.

## Problem 2

**a)** Mechanical energy is conserved.
$\frac{1}{2}kd^2 + 0 = \frac{1}{2}mv^2 + \frac{1}{2}k\left(\frac{d}{2}\right)^2, \quad mv^2 = k(d^2 - \left(\frac{d}{2}\right)^2) = \frac{3}{4}kd^2$
$v = \left(\sqrt{\frac{3k}{4m}}\right)d, \quad v = 0.87 m/s$

**b)** $X = Acos(\omega t), \quad A = 0.2\ m, \quad \omega = \sqrt{\frac{k}{m}} = 5\frac{rad}{sec}, \quad -0.1 = 0.2cos(5t), \quad cos(5t) = -0.5$
$5t = \frac{2\pi}{3}, \quad t = \frac{2\pi}{15} = 0.42s.$

**c)** $v = -A\omega sin(\omega t), \quad T = \frac{2\pi}{\omega} \Rightarrow \frac{3}{4}T = \frac{6\pi}{4\omega} = \frac{3\pi}{2\omega}$
$\omega t = \frac{3\pi}{2}, \quad sin(\omega t) = -1, \quad v = -A\omega(-1) = A\omega.$
$v = 1.0\ m/s$, to the right.
In the first $\frac{1}{4}T$, the block moves from $x_{max}$ to $x = 0$. In the second $\frac{1}{4}T$, it moves from $x = 0$ to $-x_{max}$.
In the third $\frac{1}{4}T$, it moves from $-x_{max}$ to $x = 0$ .
So at $\frac{3}{4}T$, the block is at $x = 0$, and it's moving back towards initial position.

## Problem 3

**a)** Conserve momentum:
$M_A(25)(cos(40°)) + M_B(30)(cos(25°)) = M_A(15) + M_B(v_x)$
$2M_B(25)(cos(40°)) + M_B(30)(cos(25°)) = 2M_B(15) + M_B(v_x) \;\Rightarrow v_x = 35.5m/s$
$p_y - 2M_B(25)(sin(40°)) + M_B(30)(sin(25°)) = 2M_B(0) + M_B(v_y)$
$\Rightarrow v_y = -19.5m/s.$ $\boxed{v_B = 40.5m/s \text{ @}28.8° \text{ below } x \text{ axis.}}$

**b)** $KE_I = \frac{1}{2}M_A v_{AI}^2 + \frac{1}{2}M_B v_{BI}^2 = \frac{1}{2}(2M_B)(25)^2 + \frac{1}{2}M_B(30)^2 = 1075 M_B$
$KE_F = \frac{1}{2}M_A v_{AF}^2 + \frac{1}{2}M_B v_{BF}^2 = \frac{1}{2}(2M_B)(15)^2 + \frac{1}{2}M_B(40.5)^2 = 1045 M_B$
$\boxed{KE \text{ is lost}}$

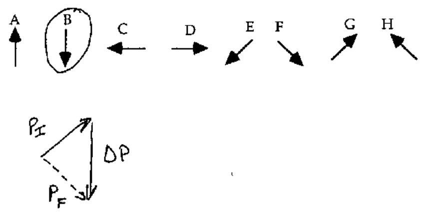
## Problem 4

**a)** For a circular orbit $\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$
$r = 2 \times 10^5 m \Rightarrow v = 113 m/s \Rightarrow \boxed{37 m/s \text{ is too slow}}$

**b)** $KE_I = \frac{1}{2}mv_I^2 \;\; PE_I = \frac{-GMm}{r_1}, \;\; KE_F = 0 \;\; PE_F = \frac{-GMm}{r_2}$
$\frac{-GM}{r_2} = \frac{-GM}{r_1} + \frac{1}{2}v_I^2, \;\; \frac{1}{r_2} = \frac{1}{r_1} - \frac{v_I^2}{2GM}, \;\; \frac{1}{r_2} = 4.73 \times 10^{-6}, \;\; r_2 = 2.11 \times 10^5 m$
$\Rightarrow \boxed{\text{height} = 1.1 \times 10^4 \; m}$

## Problem 5

**i)**




**ii)** c) The forces are equal. Newton's 3rd Law: Action/Reaction.

**iii)** c) At an angle not equal to 0° or 180°.


$P_1 = 8, \;\; P_2 = 6, \;\; P_f = (2+3)(2) = 10,$ 3-4-5 right triangle.
$\vec{P}_1 + \vec{P}_2 = \vec{P}_f$.

## Problem 6

**a)** $p_{TOT} = 14, \;\; p_{Initial} = (10)(3) - (4)(4) = 30 - 16 = 14, \;\; p_{Final} = (10)v_A + 24$
$14 = 24 + 10v_A, \;\; \boxed{v_A = -1 \; m/s = 1 \; m/s \text{ to the left}}$

**b)** $KE_I = \frac{1}{2}(10)(9) + \frac{1}{2}(4)(16) = 45 + 32 = 77$
$KE_F = \frac{1}{2}(10)(1) + \frac{1}{2}(4)(36) = 5 + 72 = 77$
$KE_I = KE_F$, collision is elastic.

**c)** $F = \frac{\Delta p}{\Delta t}, \;\; \Delta p = p_f - p_i = (4)(6) - (4)(-4) = 24 + 16 = 40$

$$\boxed{F = \frac{40}{10^{-3}} = 40,000 \; N = 40,000 \; N \text{ to the right}}$$

## Problem 7

**a)** $KE_I = 0,\ \ PE_I = \frac{1}{2}kS^2,\ \ Work = 0$
$KE_F = 0,\ \ PE_F = Mg(4S)\ ,\ \frac{1}{2}kS^2 = Mg(4S)$
$k = \frac{2Mg(4S)}{S^2},\ \ \boxed{k = \dfrac{8Mg}{S}}$

**b)**Use unstretched point as origin, and up = +.
$-ky_{eq} - Mg = 0,\ \ y_{eq} = \frac{-Mg}{k} = \frac{-Mg}{\frac{8Mg}{S}}\ ,$

$\boxed{y_{eq} = \dfrac{-S}{8}}$

**c)** $y = Acos(\omega t + \phi),\ \ v_y = 0\ \ \text{at}\ t = 0,\ \ \text{so}\ \phi = 0.$
$v_y = -Asin(\omega t + \phi).\ \ \omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{8g}{S}}$
$A = S - \frac{S}{8} = \frac{7S}{8}$

## Problem 8

$Mv_1 = 2Mv_2,\ \ v_1 = 2v_2,\ \ \frac{1}{2}(2M)v_2^2 = K,\ \ v_2 = \sqrt{\frac{K}{M}}$
$KE_{TOT} = \frac{1}{2}Mv_1^2 + \frac{1}{2}(2M)v_2^2 = \frac{1}{2}M(4v_2^2) + \frac{1}{2}(2M)v_2^2 = 3Mv_2^2 = 3M(\frac{K}{M})$
$KE_{TOT} = 3K$

## Problem 9

**a)** $V(\frac{M}{4}) = Mv - (\frac{M}{4})(\frac{V}{3})$
$Mv = (\frac{M}{4})\frac{4}{3}V,\ \ \boxed{v = \dfrac{V}{3}}$

**b)** $KE_I = \frac{1}{2}(\frac{M}{4})(V^2) = \frac{1}{8}MV^2\ ,\ KE_F = \frac{1}{2}(\frac{M}{4})(\frac{V}{3})^2 + \frac{1}{2}M(\frac{V}{3})^2 = \frac{MV^2}{9}(\frac{1}{8} + \frac{1}{2}) = \frac{Mv^2}{9}(\frac{5}{8})$
$KE_F = \frac{MV^2}{8}(\frac{5}{9}) \neq KE_I,$ $\boxed{\text{Not elastic}}$

**c)** $F = \frac{\Delta p}{\Delta t} = \frac{M\frac{V}{3} - 0}{\Delta t} = \boxed{\dfrac{MV}{3\Delta t},\ \text{to the left.}}$

**d)** Same magnitude, opposite direction. Newton's 3rd law. Action/Reaction.

## Problem 10

**a)**$(\frac{M}{4}V) = (\frac{5}{4}M)v,\ \ \boxed{v = \dfrac{V}{5}}$

**b)** Amplitude: $\frac{1}{2}kA^2 = \frac{1}{2}(\frac{5}{4}M)(\frac{V}{5})^2,\ \ \boxed{A = \dfrac{V}{5}\sqrt{\dfrac{5M}{4k}}}$

Angular frequency: $\boxed{\omega = \sqrt{\dfrac{k}{\frac{5M}{4}}} = \sqrt{\dfrac{4k}{5M}}}$

$\boxed{X = Asin(\omega t)}$

## Problem 11

**a)** $\boxed{E = \frac{1}{2}mv_0^2 - \frac{GmM_E}{R_E}}$

**b)** $\frac{1}{2}mv^2 - \frac{GmM_E}{2R_E} = \frac{1}{2}mv_0^2 - \frac{GM_E m}{R_E}$ . $\boxed{v^2 = v_0^2 - \frac{GM_E}{R_E}}$

**c)** $v = 0$ , if $\boxed{v_{0,min} = \sqrt{\frac{GM_E}{R_E}}}$

**d)** $\frac{mv^2}{2R_E} = \frac{GmM_E}{(2R_E)^2}$, $\boxed{v = \sqrt{\frac{GM_E}{2R_E}}}$

## Problem 12

**A)** $\omega_{left} = \sqrt{\frac{k}{m}}$, $\omega_{right} = \sqrt{\frac{2k}{2m}} = \sqrt{\frac{k}{m}}$

$\boxed{\text{Same } \omega \Rightarrow \text{ same period}}$ $\Rightarrow$ same time to $x = 0$.

**B)** i) Let right be positive direction: $F_{TOT} = \boxed{0 = \frac{G(5M)m}{(D-d)^2} - \frac{G(M)(m)}{d^2}}$

Two forces can cancel. Could also write $(D-d)^2 = 5d^2$

(Not required: $d = 0.31D$)

**B)** ii)$PE_{TOT} = \frac{-G(5M)m}{(D-d)} - \frac{GMm}{d}$ , $D > d$ so both terms are negative, so $\boxed{\text{it can NEVER be zero}}$.

## Problem 13

Rocket A: $E_I = \frac{-GMm}{R_E}$, $E_F = \frac{-GMm}{4R_E}$ , $W_A = \Delta E = \frac{-GMm}{4R_E} - (-\frac{GMm}{R_E}) = \boxed{\frac{3GMm}{4R_E} = W_A}$

Rocket B: $E_I = \frac{-GMm}{R_E}$ , $\frac{GMm}{(2R_E)^2} = \frac{mv^2}{2R_E}$, $v^2 = \frac{GM}{2R_E}$, $KE_F = \frac{1}{2}\frac{GMm}{2R_E} = \frac{1}{4}\frac{GMm}{R_E}$

$PE_F = \frac{-GMm}{2R_E}$, $E_F = \frac{1}{4}\frac{GMm}{R_E} - \frac{GMm}{2R_E} = -\frac{GMm}{4R_E}$ . $\boxed{W_B = \Delta E = \frac{3GMm}{4R_E} \Rightarrow \text{Same as } A\text{ .}}$

## Problem 14

**a)** $\boxed{C = \omega = \sqrt{\frac{k}{m}}}$ , $\boxed{B = V_0}$. $\frac{1}{2}kA^2 = \frac{1}{2}mV_0^2 \Rightarrow$ $\boxed{A = \sqrt{\frac{m}{k}}V_0}$

**b)** At $T/2$, block returns to $x = 0$, so $\boxed{V_{AVG} = 0 \text{ at } t = T/2}$

**c)** $V_{AVG} = \frac{\sqrt{\frac{m}{k}}V_0 sin(\sqrt{\frac{k}{m}}\frac{T}{3})}{T/3}$ . But $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$, so: $\boxed{V_{AVG} = \frac{3V_0}{2\pi}sin(\frac{2\pi}{3})}$

(Not required: $V_{AVG} = 0.4V_0$)