# Evaluating Projects under Uncertainty

March 17, 2004



- Project risk = possible variation in cash flows

Commonly used measure of project risk is the variability of the return

Variance ($\sigma^2$)

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\, dx$$

Semivariance ($S_b$)

$$S_b = \int_{-\infty}^{b} (b - x)^2 f(x)\, dx$$

Probability-of-Loss criterion

$$L = \int_{-\infty}^{b} f(x)\, dx$$

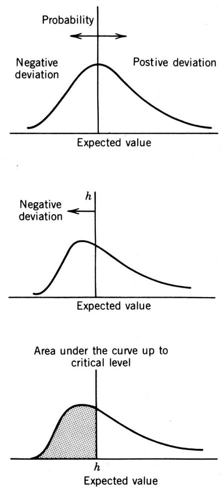



**FIGURE 10.1. Statistical measures of risk.**



# Methods of dealing with uncertainty in project evaluations

- Sensitivity analysis
- Risk adjusted MARR
- Probability trees
- Monte Carlo simulations

## Example showing how use of 'risk-adjusted' MARRs can lead to the wrong decision (from Sullivan et al, *Engineering Economy*, (11th ed), p. 445)

The Atlas Corporation is considering two alternatives, both affected by uncertainty to different degrees, for increasing the recovery of a precious metal from its smelting process. The firm's MARR for its risk-free investments is 10% per year.

| | Alternative | |
|---|---|---|
| End-of-year, k | P | Q |
| 0 | -160,000 | -160,000 |
| 1 | 120,000 | 20,827 |
| 2 | 60,000 | 60,000 |
| 3 | 0 | 120,000 |
| 4 | 60,000 | 60,000 |

Because of technical considerations, Alternative P is thought to be more uncertain than Alternative Q. Therefore, according to the Atlas Corporation's "Engineering Economy Handbook", the risk-adjusted MARR applied to P will be 20% per year and the risk-adjusted MARR for Q has been set at 17% per year. Which alternative should be recommended?



## Solution

At the risk-free MARR, both alternatives have the same PW of $39, 659.

What to do?

All else equal, choose Q, because it is less uncertain (hence less riskier) than P.

But now, do a PW analysis, using Atlas Corporation's prescribed risk-adjusted MARRs for the two options:

$PW_P$ = -160,000 + 120,000 (P/F, 20%, 1) + 60,000 (P/F, 20%, 2) + 60,000 (P/F, 20%, 4)

= $10,602

$PW_Q$ = -160,000 + 20,827 (P/F,17%,1) + 60,000 (P/F,17%, 2) + 120,000 (P/F,17%,3) + + 60,000 (P/F,17%, 4)

= $8575

Hence according to this method we would choose P. In other words, using the risk-adjusted MARRs makes the more uncertain project, P, look MORE attractive than Q!!

# Solution (contd.)

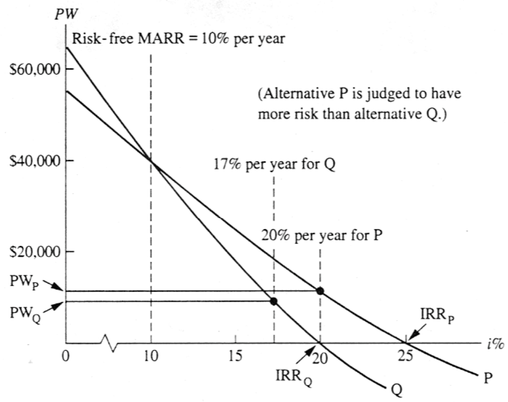



**FIGURE 10-6**
Graphical Portrayal of Risk-Adjusted Interest Rates (Example 10-8)

From Sullivan et al, p.447



# Example of probabilistic analysis

Consider the simple decision whether to make a new investment, when there is uncertainty about the duration of demand.

$I_0$ = 6800
R = 7000/yr
M = 2000 + (n-1)1000
i = 20%

| N | $I_N$ |
|---|---|
| 1 | 1600 |
| 2 | 800 |
| 3 | 400 |
| 4 | 200 |

The probability of demand for the service provided by this asset persisting for:

| | |
|---|---|
| 1yr | 0.1 |
| 2yr | 0.2 |
| 3yr | 0.3 |
| 4yr | 0.4 |

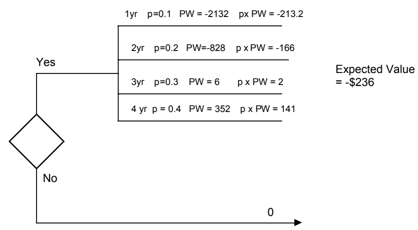
Question: Should this investment be made?

# Example (continued)

Question: Should this investment be made?

$$LAC = I_o(A/P,20\%,N) - I_N(A/F,20\%,N) + 2000 + 1000(A/G,20\%,N)$$

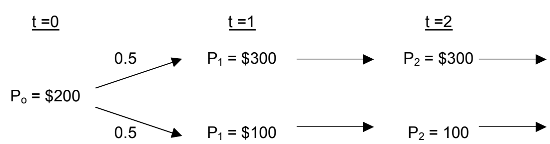
| N | LAC | Levelized OI | (P/A,20%,N) | PW of O.I. | Probability |
|---|---|---|---|---|---|
| 1 | -9560 | -2360 | 0.833 | -2132 | 0.1 |
| 2 | -7542 | -542 | 1.528 | -828 | 0.2 |
| 3 | -6997 | 3 | 2.106 | 6 | 0.3 |
| 4 | -6864 | 136 | 2.589 | 352 | 0.4 |






# The Problem of Investment Timing

Example 1: Uncertainty over Prices
Widget factory
Initial cost = I = $1600
Annual operating cost = 0
Production rate = 1 widget per year
Current widget price = $200
Price next year (and forever after):
    $300 with probability 0.5
    $100 with probability 0.5




Assume interest rate of 10%/yr

Question: Should the firm invest now, or should it wait for 1 year and see whether the price of widgets goes up or down?

## Example 1 (contd.)

Since expected price of widgets is always \$200, the NPV of an investment now is

$$NPV = -1600 + \sum_{0}^{\infty} \frac{200}{(1+0.1)^t} = -1600 + 2200 = \$600$$

Thus it might seem sensible to go ahead.

But what if we wait until next year? Then we would decide to invest only if the price goes up. If the price falls, it would make no sense to invest.

The NPV in this case is given by:

$$NPV = (0.5)\left[\frac{-1600}{1.1} + \sum_{t=1}^{\infty} \frac{300}{(1+0.1)^t}\right] = 0.5\left[\frac{-1600}{1.1} + \frac{3300}{1.1}\right] = \frac{850}{1.1} = \$773$$

So if we wait a year before deciding whether to invest in the factory, the project NPV today is \$773. Clearly it is better to wait than to invest right away.

If we had no choice, and either had to invest now or never, we would obviously choose to invest, since this would have a positive NPV of \$600. But the flexibility to choose to postpone the decision and invest next year if the market price is right is worth something. Specifically, it is worth 773-600 = \$173.

In other words, we should be willing to pay up to \$173 more for an investment opportunity that is flexible than one that only allows us the choice of investing now or never. This is the value of flexibility in this case.

Still another way of saying this is that there is an opportunity cost of investing now, rather than waiting.



## Example 2 -- Uncertainty over costs

We can consider two different kinds of cost uncertainty

a. Suppose that I = \$1600 today, but that next year it will increase to \$2400 or decrease to \$800, each with a probability of 0.5. (The cause of this uncertainty could be stochastic fluctuations in input prices, or regulatory uncertainties.) The interest rate is again 10% per year

Question: Should we invest today or wait to decide until next year?

As before, if we invest today the NPV is given by:

$$NPV = -1600 + \sum_{0}^{\infty} \frac{200}{(1+0.1)^t} = -1600 + 2200 = \$600$$

If we wait until next year, it will be sensible to invest only if the investment cost falls to \$800, which happens with a probability of 0.5. In this case the NPV is given by:

$$NPV = (0.5)\left[\frac{-800}{(1+0.1)} + \sum_{t=1}^{\infty} \frac{200}{(1+0.1)^t}\right] = 0.5\left[\frac{-800}{1.1} + \frac{2200}{1.1}\right] = \$636$$

so once again it is better to wait than to invest immediately.

# Example 2 -- contd.

a. Suppose, alternatively, that there are uncertainties over how much it is going to cost to complete the project that can only be resolved by actually doing it. You don't know for certain how much it is going to cost until you complete it. Let's say that this uncertainty takes the following form: To build the widget factory you first have to spend $1000, and that there is a 50% probability that the factory will then be complete, and a 50% probability that you will have to spend another $3000 to complete it. Assume that the widget price remains constant at $200, and that the interest rate is 10%.

At first blush, the investment would make no sense. The expected cost of the factory is:

1000 + 0.5.(3000) = 2500.

And since the value of the factory = $\sum_{t=0} \frac{200}{1.1^t} = 2200$ , we might conclude that it makes no sense to proceed. But this ignores the additional information that is generated by completing the first phase of the project, and that we can choose to abandon the project if completion requires an extra $3000. The true NPV is:

$$NPV = -1000 + (0.5)\sum_{t=0} \frac{200}{1.1^t} = +\$100$$

Since the NPV is positive, one should invest in the first stage of the project.