22.615, MHD Theory of Fusion Systems
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Lecture 11: Flux Conserving Tokamak - Con'd

A Simple Approximation

1. Instead of choosing $F(\psi)$ so $q(\psi)$ is the same everywhere, we choose a simpler $F(\psi)$ so that only $q(0)$ and $q(a)$ remain the same (as β_t increases).

2. Choose $dp/d\psi = \text{const}, dF^2/d\psi = \text{const}.$ This is the same model we have already investigated.

3. The model has only two free parameters: $A, C \rightarrow \beta_t, q_*$.

4. Thus, as β_t increases, there is only one degree of freedom, q_*, remaining.

5. Therefore, we cannot adjust q_* so that both q_0 and q_a remain fixed: this would be an overdetermined system.

6. We make an ultra simple approximation and choose q_* so that only q_a remains fixed. This prevents the formation of a separatrix which requires $q_a \rightarrow \infty$.

HBT Equilibrium

$$\mu_0 p = \beta_t B_0^2 \left(1-\rho^2\right)\left[1-\nu\rho\cos\theta\right]$$

$$B_\theta = \frac{\in B_0}{q_*}\left[\rho + \frac{\nu}{2}\left(3\rho^2 - 1\right)\cos\theta\right]$$

$$\hat{B}_\theta = \frac{\in B_0}{q_*}\left[\frac{1}{\rho} + \frac{\nu}{2}\left(1 + \frac{1}{\rho^2}\right)\cos\theta\right]$$

$$q_a = \frac{q_*}{\left(1-\nu^2\right)^{1/2}}$$

$$\nu = \frac{\beta_t q_*^2}{\in}$$

$$\rho = r/a$$

1. HBT: Express all quantities in terms of $\beta_t, q_* \sim 1/I$

2. FCT: Express all quantities in terms of β_t, q_a (held fixed). Examine the behavior as β_t increases. Are there any equilibrium limits?

Procedure

1. Define $\nu_* = \beta_t q_a^2 / \in \propto \beta_t$ since q_a is held fixed in the FCT.

2. ν_* is the heating parameter: as β_t increases, ν_* increases.

3. For the HBT: $\nu = \beta_t q_*^2 / \in \propto \beta_t$ for fixed I.

4. ν is the heating parameter for fixed I: as β_t increases, ν increases.

Relation between ν and ν_*

1. $\nu = \dfrac{\beta_t q_*^2}{\in} = \dfrac{\beta_t q_a^2}{\in}\dfrac{q_*^2}{q_a^2} = \nu_*\left(1-\nu^2\right)$

2. $\nu^2 + \dfrac{\nu}{\nu_*} - 1 = 0$

$$\nu = \frac{2\nu_*}{\left(1+4\nu_*^2\right)^{1/2}+1}$$

Compute the Physical Quantities in Terms of ν_* and Compare with the HBT

1. $I \propto 1/q_*$

   a. HBT: $\dfrac{1}{q_*} = \text{const.}$ fixed I

   b. FCT: $\dfrac{1}{q_*} = \dfrac{1}{q_a}\dfrac{1}{\left(1-\nu^2\right)^{1/2}} = \dfrac{1}{q_a}\left(\dfrac{\nu_*}{\nu}\right)^{1/2}$

$$\frac{1}{q_*} = \frac{1}{q_a}\left[\frac{1+\left(1+4\nu_*^2\right)^{1/2}}{2}\right]^{1/2}$$

2. B_v

   a. HBT: $B_v = \dfrac{\mu_0 I}{4\pi R_0}\beta_p = \dfrac{\in B_0}{q_*}\dfrac{\in\beta_p}{2} = \dfrac{\in B_0}{2}\dfrac{\nu}{q_*}$

   b. FCT: $B_v = \dfrac{\in B_0}{2}\dfrac{\nu}{q_*} = \dfrac{\in B_0}{2}\dfrac{1}{q_a}\left[\dfrac{1+\left(1+4\nu_*^2\right)^{1/2}}{2}\right]^{1/2}\dfrac{2\nu_*}{1+\left(1+4\nu_*^2\right)^{1/2}}$

$$B_V = \frac{\in B_0}{2}\frac{\nu_*}{q_a}\left[\frac{2}{1+\left(1+4\nu_*^2\right)^{1/2}}\right]^{1/2}$$

3. ρ_s

a. HBT: $\rho_s = \frac{1}{\nu}\left[1+\left(1-\nu^2\right)^{1/2}\right]$

b. FCT: $\rho_s = \frac{1+\left(1+4\nu_*^2\right)^{1/2}}{2\nu_*}\left[1+\left(\frac{2}{1+\left(1+4\nu_*^2\right)^{1/2}}\right)^{1/2}\right]$

4. Define the plasma evolution in $\beta_t - q_*$ space as β_t increases

a. HBT: $\frac{\beta_t q_*^2}{\in} = \nu$

$$q_* = \text{const.}$$

b. FCT: $\frac{\beta_t q_*^2}{\in} = \nu_*$ (1)

$$\frac{1}{q_*} = \frac{1}{q_a}\left[\frac{1+\left(1+4\nu_*^2\right)^{1/2}}{2}\right]^{1/2} \qquad (2)$$

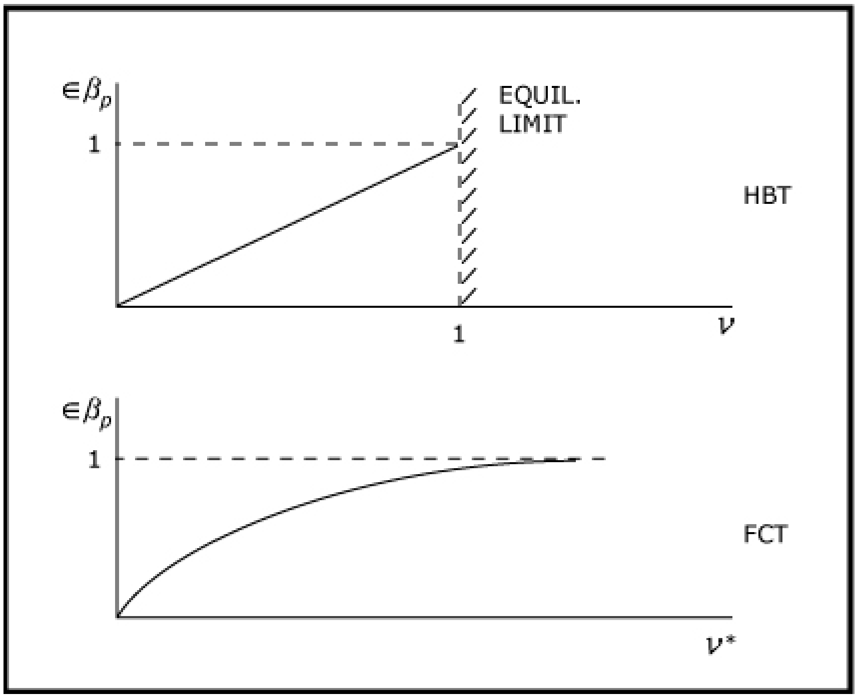
c. Solve (2) for ν_* and substitute into (1) to give $\beta_t = F\left(q_*\right)$

$$\nu_*^2 = \frac{q_a^2}{q_*^2}\left[\frac{q_a^2}{q_*^2}-1\right]$$

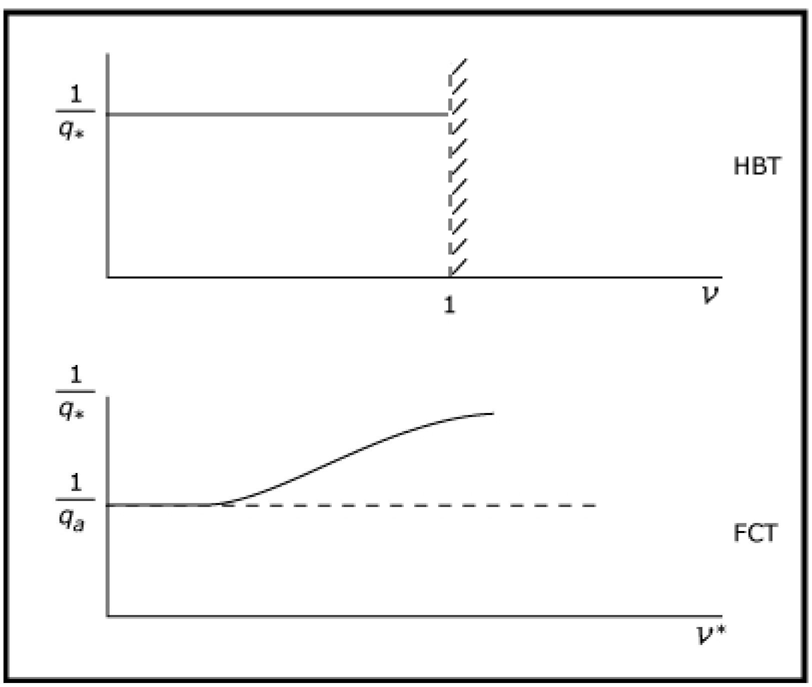
$$\frac{\beta_t q_a^2}{\in} = \left[\frac{q_a^2}{q_*^2}\left(\frac{q_a^2}{q_*^2}-1\right)\right]^{1/2}$$

Plot the Results

1.

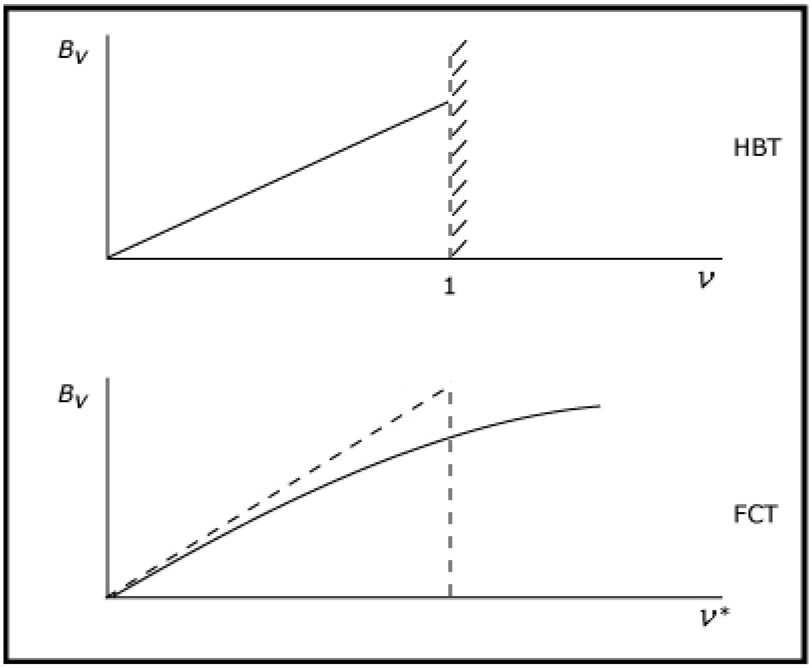


2. $I \propto 1/q_*$



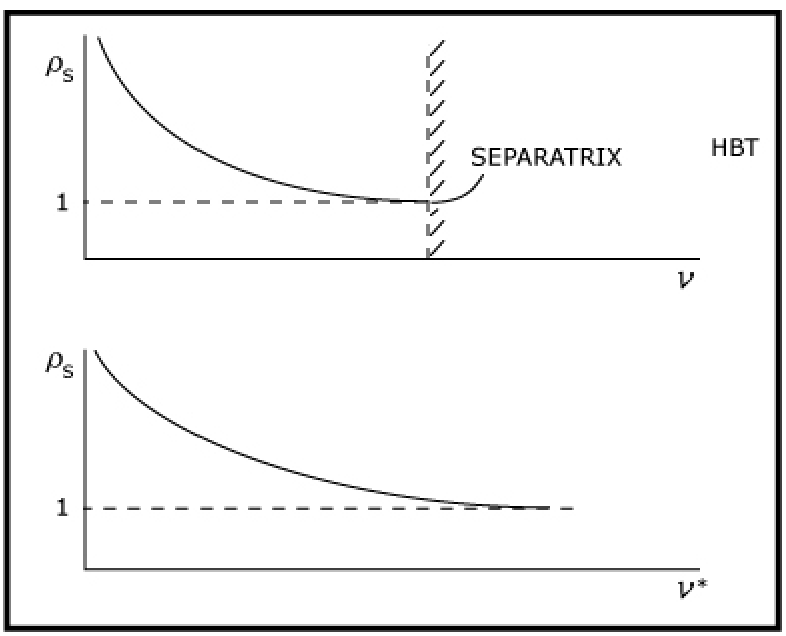
As ν_* increases, I increases. This helps to prevent the separatrix from moving onto the plasma surface since less vertical field is required to maintain toroidal force balance.

3. B_v



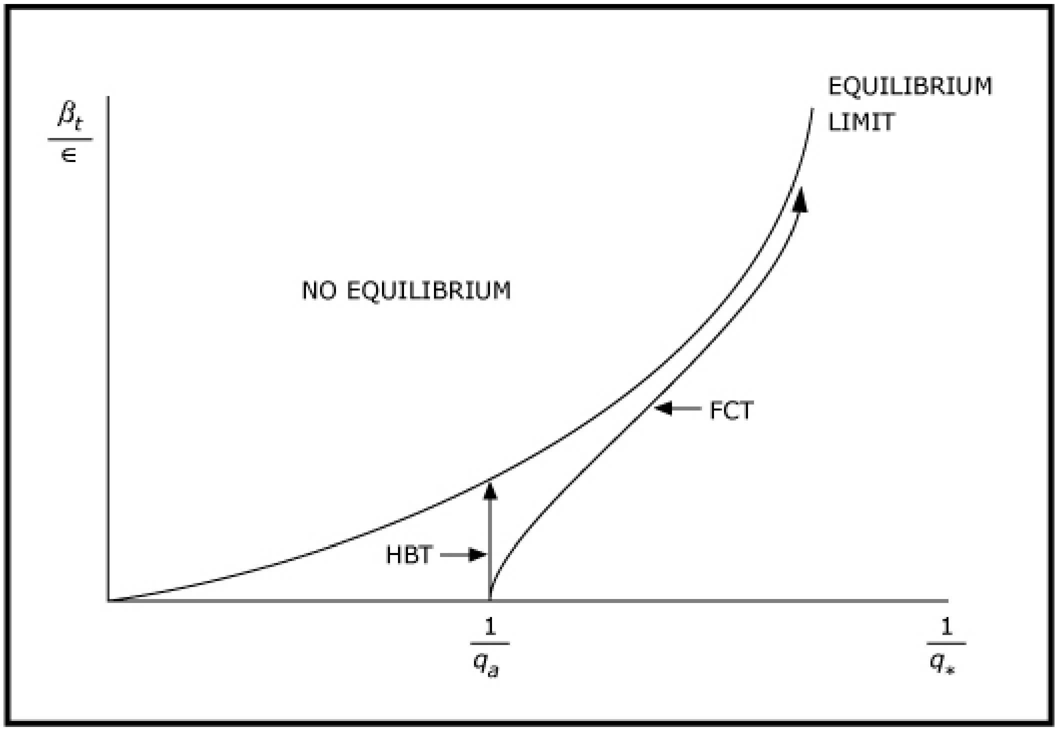
Less vertical field is required. The separatrix stays away from the plasma surface.

4. ρ_s



No equilibrium limit. The separatrix does not move onto the plasma surface.

5. β_t vs. $1/q_*$



Summary

1. General HBT: covers all permissable $\beta_t/\epsilon, q_*$ space
2. HBT at fixed I: exhibits an equilibrium limit
3. FCT at fixed q_a: no equilibrium limit