22.313 THERMAL-HYDRAULICS IN NUCLEAR POWER TECHNOLOGY

OPEN BOOK | MID-TERM QUIZ (solutions) | 1.5 HOURS

Problem 1 (20%) – Calculation of Flow Quality from Void Fraction Measurements

The information missing is the mass flux (or a superficial velocity) in the downcomer. For example, if the mass flux were known, then the following set of equations would enable calculation of the flow quality:

$$\alpha = \frac{1}{1+\frac{\rho_v}{\rho_\ell}\cdot S\cdot\frac{1-x}{x}}$$ (fundamental α-x-S equation)

$$S = \frac{v_v}{v_\ell}$$ (definition of slip ratio)

$$v_\ell - v_v = v_b$$ (relative velocity; note that in general $v_\ell > v_v$ in downflow)

$$G = \rho_v\alpha v_v + \rho_\ell(1-\alpha)v_\ell$$ (mass flux)

The unknowns are x, S, v_v and v_ℓ.

Problem 2 (30%) – Pressure Drop in Accelerating Single-Phase Flow

i) For a perfectly incompressible fluid the density ρ is constant, and so the mass and momentum equations become, respectively:

$$\frac{\partial G}{\partial z} = 0$$

$$\frac{\partial G}{\partial t} = -\frac{\partial P}{\partial z} - \frac{\partial}{\partial z}\left[\frac{G^2}{\rho}\right] - \tau_w\frac{P_w}{A} - \rho g\cos\theta \quad\Rightarrow\quad \frac{\partial G}{\partial t} = -\frac{\partial P}{\partial z} - \frac{f}{D_e}\frac{G|G|}{2\rho} - \rho g$$

where P_w, A and D_e are the channel wetted perimeter, flow area and equivalent diameter, respectively.

ii) Integrating the momentum equation with respect to z, one gets:

$$P_{inlet} - P_{outlet} = \int_0^L \frac{\partial G}{\partial t}dz + \int_0^L \frac{f}{D_e}\frac{G|G|}{2\rho}dz + \int_0^L \rho g dz$$

The pressure at the inlet is constant by assumption. The first term on the right-hand side is also constant because G increases linearly with time. The third term on the right-hand side is constant because the fluid is incompressible. The second term on the right-hand side increases roughly as t^2. Therefore, the above equation suggests that the outlet pressure must decrease roughly as t^2.

Problem 3 (50%) – Sizing of a Turbulent-Deposition Air/Water Separator

i) The Ishii-Mishima correlation gives the value of the air superficial velocity at the onset of entrainment, j_v=15.7 m/s (calculated with the thermophysical properties of Table 1). Thus the separator will have to operate at j_v=0.7×15.7 m/s ≈ 11 m/s.

Then the diameter of the separator can be calculated from the following equation:

$$j_v = \frac{xG}{\rho_v} = \frac{x\dot{m}}{\rho_v(\frac{\pi}{4}D^2)} \qquad \Rightarrow \qquad D = \sqrt{\frac{4x\dot{m}}{\rho_v \pi j_v}} = 0.196 \text{ m}$$

where x=0.95 and $\dot{m}$=0.42 kg/s.

ii) A mass balance for the water droplets in the vapor core (see notes on annular flow) gives:

$$\dot{m}(1-x)\frac{de}{dz} = -\pi D \Gamma_d$$

where 'e' is the entrained liquid fraction (e =1 at the inlet), and Γ_d is the rate of droplet deposition, which can be found as:

$$\Gamma_d = K\frac{1-x}{x}\rho_v e$$

where K=0.1 m/s is the deposition coefficient given by the McCoy-Hanratty correlation.
Integration of the mass balance equation gives:

$$e(L) = e(0)\cdot \exp(-\frac{\pi D K \rho_v}{x\dot{m}}L)$$

where L is the length of the separator. If 'e' is to decrease by 50%, then the required length is:

$$L = \frac{x\dot{m}}{\pi D K \rho_v}\log(2) \approx 3.7 \text{ m}$$

iii) The separation efficiency of the separator is 50%, since 50% of the initial moisture content is removed.