22.312 ENGINEERING OF NUCLEAR REACTORS

Thursday, November 12th, 2015

OPEN BOOK QUIZ 2 SOLUTIONS

Problem 1 (65%) – Helium-cooled fast reactor with molten fuel within steel rods

i)
The energy equation for the coolant is:

$$\dot{Q} = \dot{M}c_p(T_{out} - T_{in}) \quad \Rightarrow \quad T_{out} = T_{in} + \frac{\dot{Q}}{\dot{M}c_p} \approx 600°\text{C} \qquad (1)$$

where T_{in} = 400°C is the bulk coolant temperature, $\dot{M}$ =1440 kg/s, c_p= 5.2 kJ/kg°C, and $\dot{Q}$ = 1,500 MW.

The average linear power is $q' = \dot{Q}/(N_{pin}L)$=16.67 kW/m, N_{pin}=30,000 and L = 3 m.

ii)
Considering one rod and its coolant channel under the assumptions given in the problem statement (gravity + friction only, smooth rod surface, fully-developed flow), the total pressure drop can be expressed as:

$$-\Delta P_{tot} = f\frac{L}{D_e}\cdot\frac{G^2}{2\rho} + \rho gL = f\frac{L}{(4A/P_w)}\cdot\frac{(\dot{m}/A)^2}{2\rho} + \rho gL \qquad (2)$$

where $-\Delta P_{tot}$=120 kPa (per the problem statement), $\dot{m} = \dot{M}/N_{pin}$≈0.048 kg/s, ρ = 4.81 kg/m³, A is the unknown flow area of the channel ($A = p^2 - \frac{\pi}{4}d_{co}^2$), and P_w is the wetted perimeter $P_w = \pi d_{co}$=31.4 mm. Assuming turbulent, fully-developed flow, a smooth rod and a large Reynolds number, the friction factor can be found from the correlation

$$f = \frac{0.184}{\text{Re}^{0.2}} = \frac{0.184}{\left(\frac{GD_e}{\mu}\right)^{0.2}} = \frac{0.184}{\left(\frac{\dot{m}4}{\mu P_w}\right)^{0.2}} \qquad (3)$$

where μ = 2.8×10⁻⁵ Pa·s. Substituting Eq. (3) into Eq. (2) and solving for A, we get:

$$A = \left[0.0174\frac{\mu^{0.2}P_w^{1.2}\dot{m}^{1.8}L}{\rho(-\Delta P_{tot} - \rho gL)}\right]^{1/3} \approx 90.5 \text{ mm}^2$$

Then the rod-to-rod pitch can be found to be $p \approx 13$ mm. At this value of p the mass flux is $G \approx$ 530 kg/m^{2}s and the Reynolds number is $Re \approx$ 218,270, which confirms the accuracy of the assumption made.

iii)
Considering only conduction (per the problem statement), the radial temperature distribution within the fuel can be easily found by integrating the heat equation:

$$k_f \frac{1}{r}\frac{d}{dr}[r\frac{dT}{dr}] + q''' = 0$$

with boundary conditions dT/dr=0 at r=0 and T=T_{ci} at r=R_{ci}=R_{co}-t_c= 4.5 mm:

$$T(r) = T_{ci} + \frac{q'}{4\pi k_f}\left[1-\left(\frac{r}{R_{ci}}\right)^2\right] \quad (4)$$

where k_f = 0.7 W/m·K (assumed to be independent of temperature, as per the problem statement) and the linear power at the conditions of interest is q'=5.5 kW/m (calculated from the 500 MW core power). Setting $T(r) = T_m$ = 460°C (the fuel freezing point) in Eq. (4), and solving for r we get the radius of the molten region as follows:

$$R_m(z) = R_{ci}\sqrt{1-\frac{4\pi k_f(T_m - T_{ci}(z))}{q'}} \quad (5)$$

Now recall that the temperature on the inner surface of the cladding is:

$$T_{ci}(z) = T_b(z) + q'\left[\frac{1}{2\pi k_c}\ln\frac{R_{co}}{R_{ci}} + \frac{1}{2\pi R_{co}h}\right] \quad (6)$$

where k_c= 19 W/m°C, and $h \approx$ 6955 W/m^2°C is the heat transfer coefficient found from the Dittus-Boelter correlation, which is appropriate for the conditions of interest (non-metallic fluid, fully-developed, turbulent flow[1]). Also note that from the conservation of energy for the coolant we get:

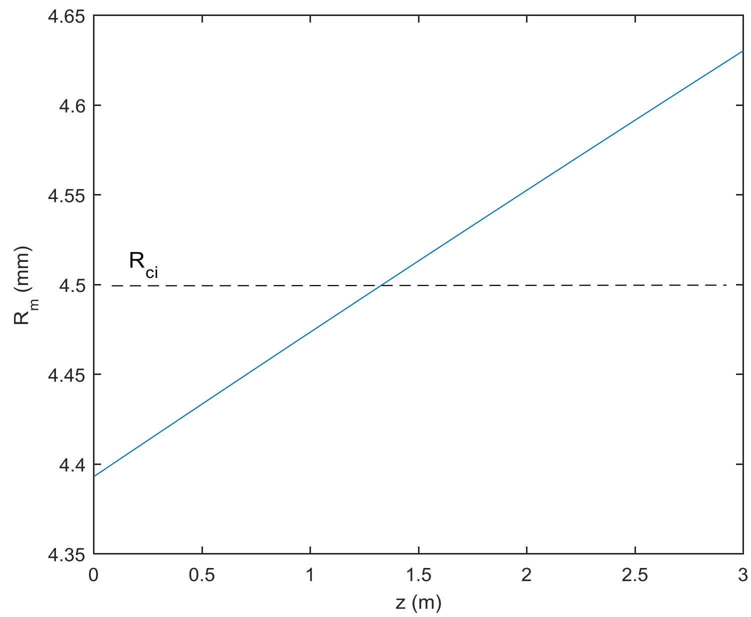
$$T_b(z) = T_{in} + \frac{q'}{\dot{m}c_p}z \quad (7)$$

Finally, back substituting Eqs. (7) and (6) into Eq. (5), we get:

$$R_m(z) = R_{ci}\sqrt{1-4\pi k_f\left\{\frac{T_m - T_{in}}{q'} - \frac{z}{\dot{m}c_p} - \left[\frac{1}{2\pi k_c}\ln\frac{R_{co}}{R_{ci}} + \frac{1}{2\pi R_{co}h}\right]\right\}} \quad (8)$$

[1] Similar values of the heat transfer coefficient are obtained using other correlations applicable to these conditions, e.g. simplified Gnielinski's or Petukhov's

which is plotted in the figure below. Note that above z = 1.33 m there is no frozen annulus in the fuel rods.



Problem 2 (35%) – Passive Residual Heat Removal System

Neglecting all pressure losses in the primary system except for the friction loss in the core, the momentum equation for the primary loop is:

$$\beta\rho(T_{c,out}-T_{c,in})gH_{pl} = f_{core}\frac{L_{core}}{D_{core}}\cdot\frac{(\dot{M}_{pl}/A_{core})^2}{2\rho} \quad (9)$$

where $T_{c,out}$ and $T_{c,in}$ are the unknown core inlet and outlet temperatures, respectively, $\dot{M}_{pl}$ is the unknown mass flow rate in the primary system, H_{pl} = 10 m, f_{core} = 0.02; A_{core} = 1 m^2; L_{core} = 3 m; D_{core} = 1.2 cm, $\beta = 2\times10^{-3}$ 1/°C and ρ = 800 kg/m^3. The energy equation for the core is:

$$\dot{Q} = \dot{M}_{pl}c(T_{c,out}-T_{c,in}) \quad (10)$$

where $\dot{Q}$ = 9 MW and c = 5.0 kJ/kg°C. Substituting Eq. (10) into Eq. (9) and solving for $\dot{M}_{pl}$, we get:

$$\dot{M}_{pl} = \left[\frac{2\rho^2\beta\dot{Q}gH_{pl}D_{core}A_{core}^2}{f_{core}L_{core}c}\right]^{1/3} \approx 448.8 \text{ kg/s}$$

Now let us move to the RHRS loop. Neglecting all pressure losses except for an equivalent form loss with K_R = 40 (per the problem statement), the momentum equation for the RHRS loop is:

$$\beta\rho(T_{R,hot} - T_{R,cold})gH_R = K_R \frac{(\dot{M}_R / A_R)^2}{2\rho} \tag{11}$$

where $A_R = \frac{\pi}{4} D_R^2$ and D_R = 15 cm, $T_{R,high}$ is the unknown hot leg temperature in the RHRS loop, $T_{R,cold}$ = 100°C (per the problem statement) and H_R = 10 m. The energy equation for the RHRS is:

$$\dot{Q} = \dot{M}_R c(T_{R,hot} - T_{R,cold}) \tag{12}$$

Substituting Eq. (12) into Eq. (11) and solving for $\dot{M}_R$, we get:

$$\dot{M}_R = \left[\frac{2\rho^2 \beta \dot{Q} g H_R A_R^2}{K_R c}\right]^{1/3} \approx 15.2 \text{ kg/s}$$

Then from Eq. (12), we can get the hot leg temperature in the RHRS loop:

$$T_{R,hot} = T_{R,cold} + \dot{Q} / (\dot{M}_R c) \approx 218.2°\text{C}$$

Finally, since $T_{c,out} - T_{R,hot}$ =10°C (per problem statement), we get $T_{c,out}$ =228.2°C, and from Eq. (10):

$$T_{c,in} = T_{c,out} - \dot{Q} / (\dot{M}_{core} c) \approx 224.2°\text{C}$$