**22.312 ENGINEERING OF NUCLEAR REACTORS**

Due November 7, 2008 by 12:00 pm

TAKE HOME QUIZ 2 (solution)

**Problem 1 (75%) – Cooling system for an accelerator target**

i)

The total beam power, $\dot{Q}$=200 W, can be obtained by integrating the volumetric heat generation over the whole volume of the tungsten target, $V_W$:

$$\dot{Q} = \iiint_{V_W} q''' dV_W = q'''_{\max} \int_0^{\delta} e^{-\alpha x} w dx \int_{-L/2}^{L/2} \cos\left(\frac{\pi z}{L}\right) dz = \frac{2 q'''_{\max} w L}{\pi \alpha}(1 - e^{-\alpha \delta})$$

From which we get:

$$q'''_{\max} = \frac{\pi \alpha \dot{Q}}{2 w L (1 - e^{-\alpha \delta})} \approx 1.28 \times 10^9 \text{ W/m}^3$$

ii)

First let us use the energy equation for the Na-K coolant, to find the axial distribution of the coolant bulk temperature, $T_b(z)$:

$$\dot{m} c_p \frac{dT_b}{dz} = q''(z) P_h \qquad (1)$$

where the heated perimeter for the Na-K coolant channel is $P_h = w$, and the heat flux into the coolant is:

$$q''(z) = \int_0^{\delta} q''' dx = \frac{q'''_{\max}}{\alpha}(1 - e^{-\alpha \delta}) \cos\left(\frac{\pi z}{L}\right)$$

Integrating Eq (1) with the initial condition $T_b(-L/2) = T_{in}$ = 50°C, we get:

$$T_b(z) = T_{in} + \frac{q'''_{\max} L w}{\alpha \pi \dot{m} c_p}(1 - e^{-\alpha \delta})[1 + \sin(\frac{\pi z}{L})] \qquad (2)$$

To get the temperature distribution within the tungsten target, we now need to solve the heat conduction equation:

$$0 = k_W \frac{d^2T}{dx^2} + q'''(x,z) \qquad (3)$$

where conduction in the y and z directions was neglected, as per the problem statement. The boundary conditions for Eq. (3) are:

$-k_W \frac{dT}{dx} = 0$ at $x = 0$ (no heat transfer at the surface exposed to the beam)

$-k_W \frac{dT}{dx} = h(T - T_b)$ at $x = \delta$ (convective heat transfer at the surface exposed to the coolant)

The heat transfer coefficient, h, can be found from the chart:

- fully developed flow, as per problem statement
- equivalent diameter $D_e = 4sw/[2(s+w)] \approx 3.33$ mm
- average Na-K velocity $V = \frac{\dot{m}}{\rho s w} \approx 0.312$ m/s
- laminar flow ($Re = \frac{\rho V D_e}{\mu} \approx 1800 < 2100$)
- $\xi = s/w = 0.2 \Rightarrow Nu \approx 4.5$
- $h = Nu \cdot k/D_e \approx 32.4$ kW/m$^2$°C

Integrating Eq. (3) with the above boundary conditions, we get the temperature distribution within the tungsten target:

$$T(x,z) = T_b(z) + \frac{q'''_{\max} \cos(\frac{\pi z}{L})}{\alpha} \left\{ \frac{e^{-\alpha\delta} - e^{-\alpha x}}{\alpha k_W} + \frac{\delta - x}{k_W} + \frac{1 - e^{-\alpha\delta}}{h} \right\} \qquad (4)$$

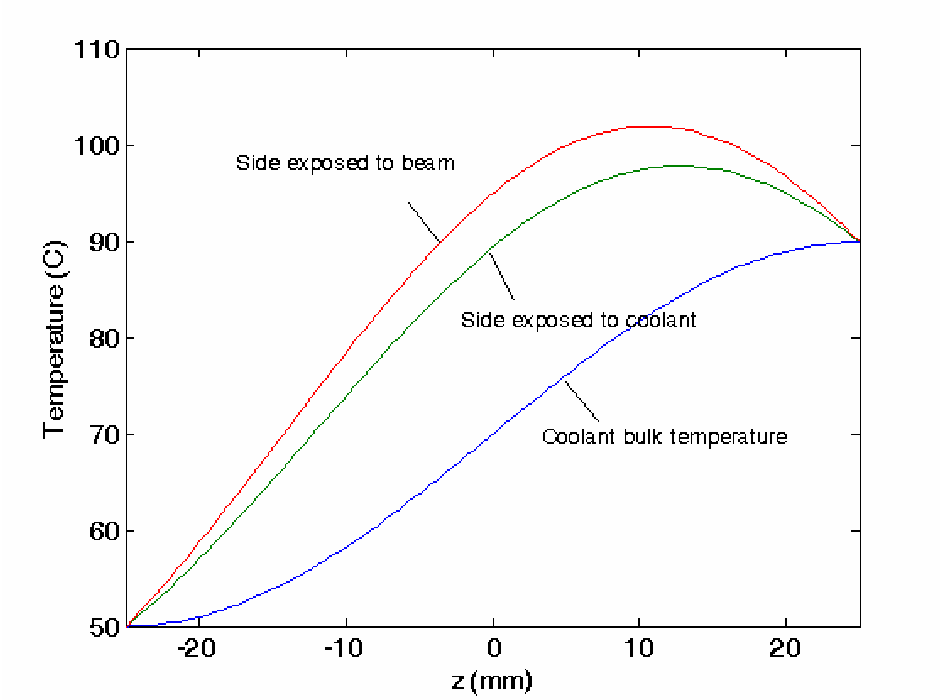
Where $T_b(z)$ is given by Eq. (2).

iii)
To find the maximum temperature in the target, we note that, at any given axial location, the temperature is maximum at the surface exposed to the beam (x = 0). Thus, we can set x = 0 in Eq. (4), differentiate with respect to z, set the derivative equal to zero, and solve for z:

$$\frac{\partial T(0,z)}{\partial z} = 0 \quad \Rightarrow \quad \therefore z_{\max} = \frac{L}{\pi} \tan^{-1}\left\{ \frac{Lw(1 - e^{-\alpha\delta})}{\pi \dot{m} c_p \left[ (1 - e^{-\alpha\delta})(\frac{1}{h} - \frac{1}{\alpha k_W}) + \frac{\delta}{k_W} \right]} \right\} \approx 10.7 \text{ mm}$$

Thus the maximum temperature occurs above the target midplane, as expected. Substituting $z_{max}$ into Eq (4), again for $x = 0$, we get $T_{max}$=101.9°C.

iv)
The coolant and target temperatures of interest are shown in the Figure below.




v)
The length of the velocity entrance region in laminar flow can be estimated as $L_v/D_e \sim 0.05 \cdot Re \Rightarrow L_v \approx 300$ mm. Therefore, the velocity profile can be assumed to have fully developed before the target area.

On the other hand, the thermal entrance region starts at the lower edge of the target, and its length (for metallic fluids in laminar flow) can be estimated as $L_T/D_e \sim 0.004 \cdot Re \Rightarrow L_T \approx 24$ mm, which is a significant fraction of the 50-mm length of the target.

In summary, the assumption of fully-developed flow made in 'ii' is not accurate, because the temperature profile develops over a significant fraction of the target region. Rigorously, one should use a heat transfer correlation that accounts for a developing temperature profile in the presence of a fully-developed velocity profile.

**Problem 2 (25%) – Natural circulation flow**

Under the assumptions recommended in the problem statement the momentum equation for the riser is:

$$P_A - P_B = \rho_H gH \tag{5}$$

where $\rho_H$ is the water density in the riser. The momentum equation for downcomer pipe # 1 is:

$$P_B - P_A = -\rho_C gH + f_1 \frac{H}{D_1} \cdot \frac{G_1^2}{2\rho_C} \tag{6}$$

Where $\rho_C$ is the water density in the downcomer, $D_1$ is the diameter of pipe #1, $G_1$ is the mass flux in pipe #1 and $f_1$ is the friction factor in pipe #1 (found from the MacAdams correlation):

$$f_1 = \frac{0.184}{\mathrm{Re}^{0.2}} = \frac{0.184}{(G_1 D_1 / \mu)^{0.2}} \tag{7}$$

Substituting Eq. (7) into Eq. (6), eliminating $P_A$-$P_B$ from Eqs. (5) and (6), and solving for $G_1$, we get:

$$G_1 = \left( \frac{2\rho_C^2 g\beta D_1^2 \Delta T}{0.184\mu^{0.2}} \right)^{1/1.8} \approx 1002 \text{ kg/m}^2\text{s}$$

where the Boussinesq's approximation was used to find $\rho_C - \rho_H = \rho_C\, \beta\, \Delta T$, with $\Delta T$=30°C.

Once $G_1$ is known, the mass flow rate in pipe #1 is readily found, $\dot{m}_1 = G_1 \frac{\pi}{4} D_1^2$ = 7.87 kg/s.

Similarly, for pipe #2:

$$P_B - P_A = -\rho_C gH + f_2 \frac{H}{D_2} \cdot \frac{G_2^2}{2\rho_C} \quad \Rightarrow \quad G_2 = \left( \frac{2\rho_C^2 g\beta D_2^2 \Delta T}{0.184\mu^{0.2}} \right)^{1/1.8} \approx 631 \text{ kg/m}^2\text{s}$$

And $\dot{m}_2$ = 1.24 kg/s. Finally the total mass flow rate in the loop is $\dot{m}_{tot} = \dot{m}_1 + \dot{m}_2$ = 9.1 kg/s.

Note that for the calculated values of $G_1$ and $G_2$, the Reynolds numbers in pipes #1 and 2 are ~125000 and ~40000, respectively. So the assumption of turbulent flow is accurate.

22.312 Engineering of Nuclear Reactors
Fall 2015