22.312 ENGINEERING OF NUCLEAR REACTORS

Tuesday, December 15th, 2015, 9:00 a.m. – 12:00 p.m.

OPEN BOOK FINAL EXAMS (SOLUTIONS)

Problem 1 (35%) – Containment Heat Transfer and Structural Mechanics

i)
Heat rejection from the containment can be viewed as the series of three heat transfer processes:

$$\dot{Q} = Ah_{in}(T_{in} - T_{ci}) \quad \text{(condensation on the inner shell)} \quad (1)$$

$$\dot{Q} = \frac{Ak_c}{t_c}(T_{ci} - T_{co}) \quad \text{(conduction through the shell)} \quad (2)$$

$$\dot{Q} = Ah_{out}(T_{co} - T_{out}) \quad \text{(convection to air on the outer shell)} \quad (3)$$

where A = 50,000 m^2, t_c = 25 mm and k_c=40 W/m°C are the is the containment surface area, thickness and thermal conductivity, respectively; h_{in} = 1,000 W/m^2°C and h_{out} = 5 W/m^2°C are the steam condensation and air convection heat transfer coefficients, respectively; T_{out} = 30°C and T_{in} = 150°C are the ambient temperature and the temperature inside the containment, respectively; T_{ci} and T_{co} are the (unknown) temperatures on the inner and outer surfaces of the containment shell, respectively. Eliminating T_{ci} and T_{co}, and solving for the heat rejection rate $\dot{Q}$, we get:

$$\dot{Q} = \frac{T_{in} - T_{out}}{\frac{1}{Ah_{in}} + \frac{t_c}{Ak_c} + \frac{1}{Ah_{out}}} \approx 29.8 \text{ MW} \quad (4)$$

ii)
It would NOT make sense to change containment material to a higher thermal conductivity material, because the thermal resistance associated with conduction within the containment ($t_c/(Ak_c)$) is already very small, thus the impact on heat rejection rate would be negligible. Efforts should focus on increasing the heat transfer coefficient on the outer surface of the containment shell, which is the dominant thermal resistance. This could be accomplished, for example, by spraying the shell with water to induce evaporative cooling, which is the approach used in the AP1000 design.

iii)
The total pressure at the final state (State 2) is:

$$P_2 = P_{w2} + P_{a2} \quad (5)$$

Where P_{w2} = 476 kPa is the partial pressure of the dry saturated steam at T_2 = 150°C. Since the volume and mass of air in the containment have not changed, the ideal gas equation of state provides the partial pressure of air at state 2 as:

$$P_{a2} = P_{a1}\, T_2 / T_{a1} \approx 141 \text{ kPa} \qquad (6)$$

where P_{a1} = 101 kPa and T_{a1} = 303 K (30°C) are the initial (atmospheric) pressure and temperature in the containment, respectively. Substituting the numbers back into Eq. 5, we find $P_2 \approx$ 617 kPa.

iv)
The total mass of steam in the containment at State 2 is:

$$M_w = V_c / v_g \approx 191{,}000 \text{ kg} \qquad (7)$$

where V_c = 75,000 m^3 is the containment volume and v_g = 0.39 m^3/kg is the specific volume of saturated steam at 150°C.

v)
The thin shell theory can be used since R_c / t_c = 800 >10, where R_c = 20 m and t_c = 25 mm. Then the hoop stress is

$$\sigma_\theta = (P_{in} - P_{out}) R_c / t_c \qquad (8)$$

Solving Eq. 8 for P_{in} and setting σ_θ = 500 MPa, we find $P_{in} \approx$ 726 kPa.

Problem 2 (40%) – Creating a Two-Phase Mixture from Flashing of Pressurized Water

i)
The throttle reduces the pressure of the fluid from 10 MPa to 6 MPa. Taking the throttle as the control volume, the conservation of mass and energy yield, respectively:

$$0 = \dot{m}_{in} - \dot{m}_{out} \qquad \Rightarrow \qquad \dot{m}_{in} = \dot{m}_{out} = \dot{m}$$
$$0 = \dot{m}h_{in} - \dot{m}h_{out} \qquad \Rightarrow \qquad h_{in} = h_{out} = h_f(1-x_0) + h_g x_0 \tag{9}$$

where $\dot{m}$ = 10 kg/s is the mass flow rate, and we assumed steady-state, adiabatic flow with negligible changes in the kinetic and gravitation terms, as usual; h_{in} = 1343 kJ/kg (given in the problem statement), h_f = 1213 kJ/kg and h_g = 2785 kJ/kg are the specific enthalpies of the pressurized water at the throttle inlet and saturated liquid and vapor at the throttle outlet, respectively. We also assumed the vapor and liquid phases are in thermal equilibrium at the exit of the throttle, per the problem statement. From Eq. 9 we can readily get the steam quality in the channel, x_0:

$$x_0 = (h_{in} - h_f)/h_{fg} \approx 0.083$$

ii)
To identify the flow regime with the map provided in the problem statement, we need to first find the superficial velocities:

$$j_v = \frac{x_0 G}{\rho_g} \approx 3.43 \text{ m/s}$$
$$j_\ell = \frac{(1-x_0)G}{\rho_f} \approx 1.54 \text{ m/s}$$

where $G = \frac{\dot{m}}{A} \approx 1273$ kg/m^{2}s is the mass flux, $A = \frac{\pi}{4}D^2 \approx 78$ cm^2 is the flow area, D = 10 cm is the tube diameter, ρ_f = 758.5 kg/m^3 and ρ_g = 30.6 kg/m^3. Then the flow map suggests the flow regime is annular flow.

iii)
We resort here to a simple drift flux model: per the textbook (page 628) at high void fraction one can take C_0 = 1 and in annular flow Ishii proposed $V_{vj} \approx 0$. Thus the drift flux model effectively reduces to HEM for which:

$$\alpha = \frac{1}{1 + \frac{\rho_g}{\rho_f} \cdot \frac{1-x_0}{x_0}} \approx 0.69$$

By comparison the EPRI correlation (which is more cumbersome to use, especially during the final exam ☺) predicts $\alpha \approx 0.62$.

The friction pressure drop in the channel is:

$$\Delta P_{fric} = \phi_{\ell o}^2 f_{\ell o} \frac{L}{D} \frac{G^2}{2\rho_f} \tag{10}$$

where L = 3 m is the length of the unheated section of the tube. For $\phi_{\ell o}^2 f_{\ell o}$ we make use of the Friedel correlations (Eqs. 11.99 through 11.101b in the textbook), which is valid for all flow regimes in both vertical and horizontal channels. At the conditions of interest the parameters of the Friedel correlation are $E \approx 0.962$, $F \approx 0.140$, $H \approx 11.67$, $Fr \approx 25.2$ and $We \approx 3.14\times10^4$, $f_{\ell o} \approx 0.0112$. Substituting into Eq. 10 we get $\phi_{\ell o}^2 \approx 4.15$ and $\Delta P_{fric} \approx 4.96$ kPa.

iv)
Since the heat flux is axially uniform, dryout would occur first at the channel outlet. The critical quality at the outlet is found to be $x_{cr} \approx 0.1562$ from the CISE-4 correlation with L_b = 13 m (equal to the distance from the inlet of the tube, per the hint in the problem statement), and the coefficients $a \approx 0.6724$ and $b \approx 14.98$, calculated for P = 6 MPa, P_c = 22.1 MPa, G = 1273 kg/m^{2}s < $G^* \approx 1306$ kg/m^{2}s, $P_h = \pi D/2 \approx 15.7$ cm (the heated perimeter) and $P_w = \pi D \approx 31.4$ cm (the wetted perimeter). Then the critical power can be calculated from the conservation of energy:

$$\dot{Q}_{cr} = \dot{m}\left[x_{cr}h_g + (1-x_{cr})h_f - h_{in}\right] \approx 1156 \text{ kW}$$

The actual power in the channel is $\dot{Q} = q'' P_h L_h \approx 283$ kW, where q'' = 600 kW/m^2 and L_h = 3 m are the applied heat flux and length of the heated test section, respectively; therefore, there is a good margin to dryout in the channel (CPR = 1156/283 ≈ 4.09)

v)
The second law of thermodynamics for the throttle (steady-state, adiabatic flow) yields:

$$0 = \dot{m}s_{in} - \dot{m}s_{out} + \dot{S}_{gen} \quad \Rightarrow \quad \dot{S}_{gen} = \dot{m}(s_{out} - s_{in}) = \dot{m}[s_f(1-x_0) + s_g x_0 - s_{in}] \approx 1.4 \text{ kW/K}$$

where s_{in} = 3.23 kJ/kg-K (given in the problem statement), s_f = 3.0 kJ/kg-K and s_g = 5.9 kJ/kg-K are the specific entropies of the pressurized water at the throttle inlet and saturated liquid and vapor at the throttle outlet, respectively. The above result confirms that irreversibilities take place in the throttle.

Problem 3 (25%) – Sizing the fuel pin of a Liquid Metal Fast Breeder Reactor (LMFBR)

i)
The temperature drop, ΔT, within a solid fuel pellet can be calculated as follows (see derivation in the textbook):

$$\Delta T = \frac{q'}{4\pi k_f} \tag{11}$$

where k_f is the fuel thermal conductivity (assumed independent of temperature) and q' is the linear power. The linear power is related to the specific power, q'''_m, as follows:

$$q' = q'''_m \rho_f \frac{\pi}{4} D^2 \tag{12}$$

where ρ_f is the fuel density and D is the fuel pellet diameter. Substituting Eq. 12 into Eq. 11, solving for D, and assuming the temperature drop is constant, we can readily get the ratio of the fuel pellet diameters for the upgraded and present core:

$$D_2 = D_1 \sqrt{\frac{q'''_{m1}}{q'''_{m2}}} \approx 0.395 \text{ cm}$$

where $D_1 = 0.51$ cm is the fuel pellet diameter in the present core, $q'''_{m,1}$=120 W/g and $q'''_{m,2}$=200 W/g are the specific powers in the present and upgraded core, respectively.

ii)
The temperature drop, ΔT, within an annular fuel pellet can be calculated as follows (see derivation in the textbook):

$$\Delta T = \frac{q'}{4\pi k_f}\left[1 - \frac{\ln(D/D_v)^2}{(D/D_v)^2 - 1}\right] \tag{13}$$

where D_v is the diameter of the hole in the middle of the pellet. In an annular pellet we have:

$$q' = q'''_m \rho_f \frac{\pi}{4}(D^2 - D_v^2) \tag{14}$$

Substituting Eq. 14 into Eq. 13, assuming the temperature drop and pellet diameter are constant, we get one equation in the only unknown D_v:

$$\ln(D/D_v)^2 + 1 - \frac{q'''_{m2} - q'''_{m1}}{q'''_{m2}}(D/D_v)^2 = 0 \tag{15}$$

Solving Eq. 15 numerically (not required for full credit), we get $D_v \approx 0.185$ cm.

iii)
Solving for k_f in Eq. 11, assuming the temperature drop and the pellet diameter to be constant, and setting the ratio $\rho_{f2}/\rho_{f1} = 1.2$, we get:

$$\frac{k_{f2}}{k_{f1}} = \frac{q'''_{m2}}{q'''_{m1}} \frac{\rho_{f2}}{\rho_{f1}} = 2$$