Homework Assignment #3

22.105
Electromagnetic Interactions

Fall 2005

Distributed: Thursday, October 5, 2005

Due: Tuesday, October 17, 2005

Problem 1

A circular loop of wire has a major radius R_0, a minor radius a, and carries a current I.

a. Prove that in the limit $a \ll R_0$ the vector potential at an arbitrary observation point R,ϕ,Z is given by

$$\mathbf{A} = \mathbf{e}_\phi \frac{\mu_0 I}{\pi}\left(\frac{R_0}{R}\right)^{1/2}\frac{1}{k}\left[\left(1-\frac{k^2}{2}\right)K(k)-E(k)\right]$$

$$k^2 = \frac{4R_0R}{\left(R_0+R\right)^2+Z^2}$$

b. Calculate B_Z at the center of the loop $R=0, Z=0$.

c. Calculate the inductance of the loop assuming $a \ll R_0$. Note, even though a is small you cannot set it equal to zero.

Problem 2

A hollow metallic cylinder of radius R_0 and finite length L carries a current density

$$\mathbf{J} = (I/L)\delta(R-R_0)\mathbf{e}_\phi \qquad -L/2 \le Z \le L/2$$

Calculate the longitudinal magnetic field $B_Z(0,Z)$ along the axis.